E-Democracy's outputs in a Mamdani Fuzzy Inference System

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Abstract. This paper provides a model of digital democracy or E-democracy that relies on a Mamdani fuzzy inference system with three inputs and an output. Representative democracy has some important pillars: justice, parliament, government and president/constitutional monarch. We need another important instrument in order to build our model of E-democracy: citizenry. Thus, the inputs of our E-democracy's model are Citizenry, Justice and Delegates, where the latter includes the institutions of the representatives in democracy: parliament, government and president/constitutional monarch. We briefly introduce an input - system (processes) - output model of digital democracy that follows the pattern of a self-adaptive neural network. Using fuzzy logics, we discuss E-democracy's outputs through different approaches, with respect to some variables that describe our model.

Keywords: E-democracy, Mamdani fuzzy inference system, fuzzy sets, fuzzy logics, uncertainty

1. Introduction

We have already presented, using a Mamdani fuzzy inference system (MFIS), a more elaborated model of E-democracy, with five inputs and an output [1]. The herein paper describes a more concise model of E-democracy which has three inputs: Citizenry, Justice and Delegates and one output: E-democracy. We also briefly discuss an algorithm (AOE) that finds a minimum or a maximum output in an MFIS. Using AOE, largely discussed with mathematical formalizations [2], we extract here some conclusions based on an analysis of variables that describe E-democracy's model.

Prior to presentation of the model, we need to explain our choices for several assumptions: the fuzziness of the inputs and the output, the use of an MFIS in a sociopolitical model, the role of uncertainty in our model and the contribution of this research to innovation in general.

We start by discussing the role of uncertainty in politics and especially from a democratic point of view. Justice has been already seen for centuries as the foundation of democracy [3, 4], so that the way we understand the concept of justice should be fundamental to framing democracy. There is no wonder that so many philosophers and political thinkers tried to establish a pattern, a prototype of justice that encompasses most if not all human needs in a democratic society. But even contemporary opposite approaches in creating an optimum distributive justice, i.e. concerning allocation of goods and burdens in society, like Rawls's justice as fairness [5] or Nozick's justice of entitlement [6], do receive the same criticism. It has been well put by Sen that these were merely 'totalist' approaches [7], while justice should be subject to renewal and improvement based on continuous deliberation. Thus, an ideal democratic society [8] should be developed on participation, deliberation and inclusion (PDI) based on the only one conviction: uncertainty. Not even PDI is crucial on a long term, but we believe that it is required now. Although this may seem a post-modernist relativism that will bring nothing but some degree of chaos, the contractualist approach must not be excluded (e.g. PDI), only that for a short-medium term (this term is also subject to uncertainty). Briefly put, democracy is a trial and error

process, where the inputs are subject to permanent medium-long term innovation, meaning that they may change their initial values or they may be replaced by other inputs. Uncertainty and fuzziness are not new concepts in social sciences, and may be well-suited for problems that search for a good solution rather than the best one [9]. An emphasize on vagueness is put in studying social systems [10], while the liaison of post-modernism and control engineering is given by the fuzzy logic ability to cope with uncertainty and also by the fuzzy sets representation of human reasoning subject to knowledge uncertainty [11]. Ragin's proposal of fuzzy sets applied in social sciences preserves both the quantitative and the qualitative dimensions, seen as the formalization of the Weber's ideal type [12], a concept that may support our goal for inclusion as a key for democratic society.

We have just described uncertainty from a sociopolitical point of view and as a desideratum of democracy. But how does this cope with our fuzzy model of democracy? We might instead take uncertainty for partial belief or gradual truth. But it is more likely, as we will see in the next sections, that we refer to uncertain gradual truth, merely a combination of the former two [13]. This means that we build our fuzzy system taking into account some degree on the truth scale and some partial belief on plausibility scale. More, Dubois and Prade point up on the confusion between gradualness and uncertainty, also emphasizing the role of bipolarity on human reasoning extended with a third landmark of neutrality. But, they also describe the concept of epistemic uncertainty related to partial or incomplete information [14]. And, if we enlarge our view from individual to global scale, bipolarity extended with neutrality also expand to a set of possible values that constitute the epistemic uncertainty. We may see many individual singleton instances of neutrality as an extended bipolarity "error" propagation yielding multi-fold neutrality, if instances of individual neutrality are not all a given constant value, e.g. 0.5; otherwise extended bipolarity is a crisp three-fold polarity. For Jamieson, uncertainty, an epistemological problem and corrosive to scientific authority, is related to lack of information but also to broad cultural processes. Requiring particular contexts and social conditions, uncertainty is located in the data of the model and not in the model itself and it cannot be overcome by application of more and better science [15].

For now on, we will simply call the epistemic uncertainty or the uncertain gradual truth as uncertainty. We will propose a method to calculate it or to find a proxy for it and we will use it to build fuzzy membership functions (MFs) of the inputs and the output of our model of E-democracy. We see the fuzzy inputs, i.e. Citizenry, Justice and Delegates(CDJ), as expressions of levels of participation in the democratic process. The levels of deliberation and inclusion, the other two components of the incipient crucial PDI, are established through the fuzzy rules of our model. We will create any MF by starting from a crisp interval that will be expanded with a given uncertainty, which is the same for all fuzzy sets of the model. The uncertainty has the role of preserving some plausibility and degree of truth when identifying the possible levels of the inputs and the output on a given scale. While the uncertainty is important, the choice for crisp values is not, and we will try to prove this statement by describing the output relativity to inputs. The dependency of the output on the inputs is linear when it comes to the choice of crisp values of MFs, which is not the case for uncertainty; see section 5.

The choice for an MFIS that stands behind E-democracy's model may be surprising, as MFIS is rather designed for fuzzy logic controller that deals mostly with technological problems that are formalized in a linguistic style. But MFIS is probably the most well-known approach and very suitable for ill-defined problems where a standard mathematical formalization is almost impossible, like in our case. And some attempts to expand MFIS to problems that deal with linguistic values in a non-technological process have already been made [16, 17]. The choice for an MFIS may be summarized this way: an acknowledged approach for a new type of matter (in theory), where the benefits of a fuzzy controller should apply to a possibilistic problem (in practice).

The goal of this paper is to describe some rules of a model of E-democracy based on theoretical PDI and three quasi-practical inputs and to see what the results are when these inputs take different values. This is somehow very similar to a sensitivity analysis, a technique that deals with uncertainty, very common in economics and in finances in particular. We will try to draw some conclusions to see if we identify new or old common beliefs or patterns when fuzzy logic is applied to a democratic political system.

This paper is organized as follows: the theoretical model of E-democracy is described in section 2, fuzzy sets and fuzzy logics of E-democracy are discussed in section 3, section 4 briefly presents AOE and its components, the analysis of E-democracy's outputs is provided in section 5 and the last section concludes this article.

2. E-democracy's model

E-democracy (Electronic-Democracy) is the social form of organizing knowledge society and it is not a political regime at most, but a way of living. Looking at E-democracy as a participative democracy intermediated by digital instruments, we must firstly define democracy. Aristotle considered democracy, a people's political regime frequently subject to tyranny of majority, as a degenerated form of politeia, the theoretical constitutional political regime [18]. While Aristotle was right two millenniums ago, we nowadays regard democracy as Aristotle's politeia which is designated, in contemporary society, as (state subject to the) rule of law or *l'état de droit*. The rule of law is a political regime, but we consider democracy having become more than this, as a bottom-up built social structure, a new religion of knowledge society, which has the features of an ancient religion - the most important institution of human kind. We regard this religion as a source of citizenry and solidarity, being the collective consciousness of all individual consciousnesses [19]. More, comparing with Durkheim's religion, E-democracy is origin and goal for all sciences, while its moral communities are information and communication technology (ICT) platforms of inclusion and deliberations. If Aristotle put the state above the citizen and regarded democracy as a top-down structure, we look at democracy as a perpetual strengthening of society's institutions through the freely consent and benevolent participation and deliberation of its citizens. The main political institutions of representative democracy or of the rule of law are parliament, executive (government and presidency/constitutional monarch) and justice.

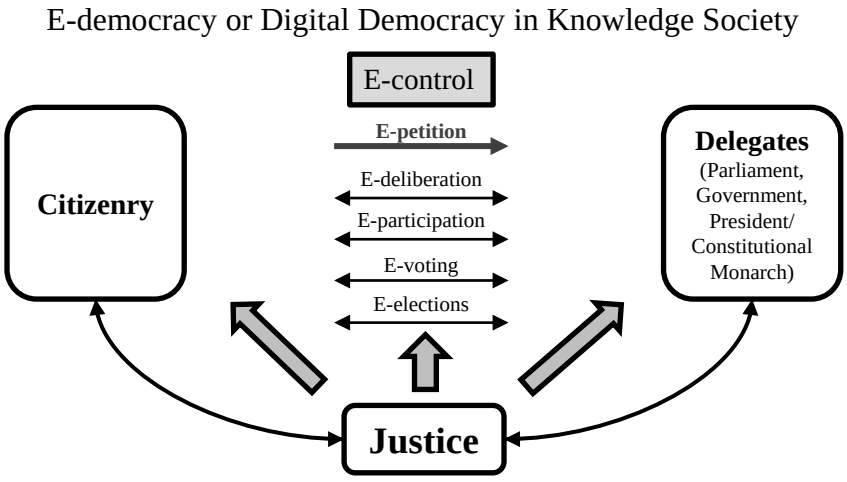

Figure 1. E-democracy's instruments

Although Sartori is against participative or any form of electronic democracy and is a supporter of liberal representative democracy [20], we must agree with him when he designates justice as the most important institution of liberal society. In our simplified model of E-democracy, we consider citizenry,

besides justice and delegates (parliament, govern and presidency/monarchy), as the second most powerful institution of democracy.

Figure 1 illustrates the relation between Citizenry, Delegates and Justice, as long with other instruments of E-democracy and it relies on the model developed by Maier [21].

In the model illustrated in Figure 1, we bring an important amendment to Maier's model by introducing e-Petition, an instrument that gives more authority to citizens. Thus, e-Petition becomes an institution of E-democracy that has some attributions: legislative proposals, legislative amendments and consultative initiatives. E-Control groups different e-instruments in such an institution that allows Citizenry to permanently and directly influence Delegates. Although a good control of Delegates by Citizenry is imperative, the watcher of E-democracy is Justice, the most important institution of any society and state subject to the rule of law.

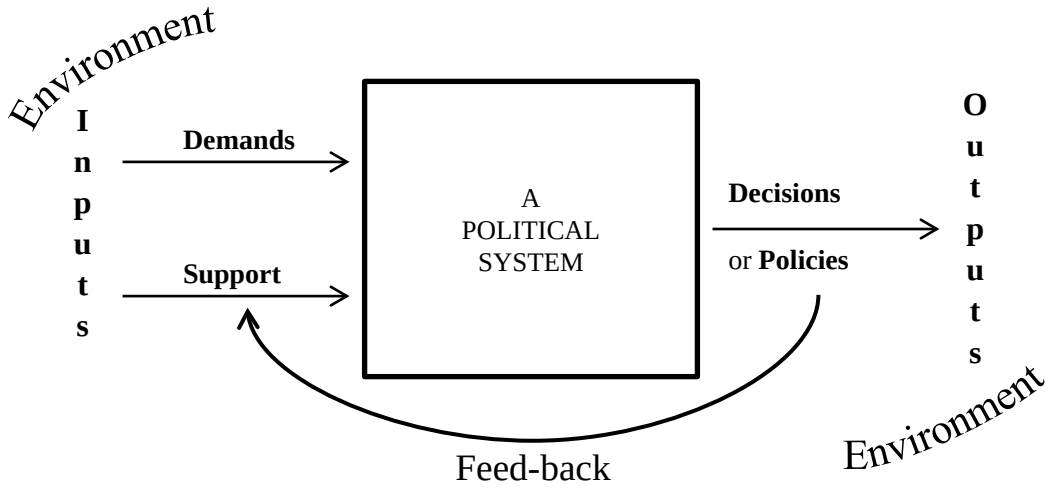

Figure 2. Easton's model of political system

An economic perspective on a political system has already emerged fifty years ago [22] and Figure 2 illustrates this input - system (processes) - output (ISPO) approach, already introduced in other explorations [1, 8].

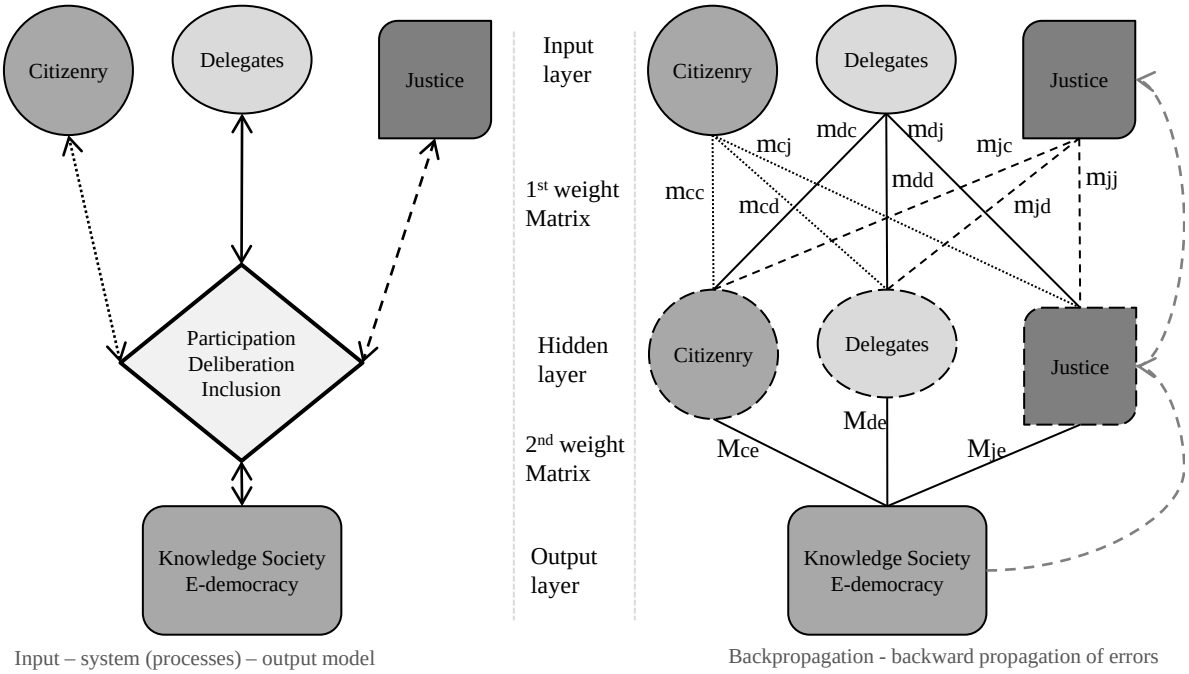

Figure 3. E-democracy as a system based on permanent optimization

Our ISPO model of E-democracy based on knowledge society (MEDKS) is a self-adaptive system that seeks for permanent optimization based on PDI. Using a parallel with the well-known artificial intelligence method of backpropagation (BPE), see Figure 3, we describe the processes that MEDKS uses to improve itself in Table 1.

Table 1. MEDKS following BPE steps to self-optimizing

Process	MEDKS	BPE
Step 1.	Planning the objectives and the margin for the optimum output	Take the (adaptive) learning rate q and predefine the maximally allowed, or desired, error E_{des}
Step 2.	Defining CDJ's tasks (weights)	Initialize weights matrices
Step 3.	Training process of PDI	Perform the on-line training patterns $p=1,..,P$
Step 4.	Checking the intermediary results	Consecutively calculate the outputs from the hidden and output layer neurons
Step 5.	Negotiations after verifying the intermediary margin of the output	Find the value of the sum of errors square cost function E_p
Step 6.	Checking the errors given by intermediary results	Calculate the output layer neurons' error signals
Step 7.	Checking the errors given by CDJ	Calculate the hidden layer neurons' error signals
Step 8.	Planning new objectives closer to the anticipated optimum output	Calculate the updated output layer weights, using learning rate q
Step 9.	Planning new tasks for CDJ	Calculate the new hidden layer weights (using q)
Step 10.	If intermediary objectives are far from the final ones, go to Step 3	If $p < P$, go to step 3. Otherwise go to step 11
Step 11.	An epoch of negotiations and learning is done. If all objectives are reached, stop the process; other way, go to Step 3	The learning epoch is completed, $p = P$. For $E_p < E_{des}$, terminate learning. Otherwise go to step 3 and start a new learning epoch, $p = 1$

In Figure 3 and Table 1, we try to import a digital model that is not perfect (but perfectible) to the real world, while most of the time it has worked the other way around (simulating real life processes with computational means). The key for MEDKS is PDI, advocated under several formalizations from ancient [18] to modern [23] and contemporary [7, 24] times. PDI is similar to the learning process through multiple training, only that for contextual problems that reach some level of clarification and help (as a groundwork) to solving subsequent issues.

We no further insist on E-democracy's model from political conceptual point of view (i.e. MEDKS), having provided a more elaborated conceptualization in a previous exploration [8]. This section has presented a computational-economic view on E-democracy that relies on inputs (i.e. CDJ), processes (i.e. PDI) and outputs (i.e. E-democracy itself). In the next section, we introduce a fuzzy model of E-democracy that defines some general rules and finds the outcomes based on these rules, using a sensitivity analysis (meaning herein to verify the role of some uncertainties in understanding the input-output relationship).

3. Fuzzy model of E-democracy (FME)

FME is practically a proposal for PDI regulation and it encompasses the inputs and outputs of MEDKS when they are subject to uncertainty. Firstly, we define the inputs and outputs from a fuzzy logic perspective. Secondly, we propose the rules that should govern FME and, finally, we argue for some choices when defining the logical structure of the model.

Although fuzzy logic approach has already made its entry in the field of social sciences, most of the researches dealt with comparative politics [25], social choice preferences [26] or decision making problems [27]. Still, fuzzy sets and fuzzy logics have been used in political systems analysis [28, 29, 30].

Should we not abandon classical logic or other types of multi-valued logics, nor the established statistic and probabilistic approaches, we believe that modeling politics might rely on more fuzzy paradigms. In this paper, we are interested in defining relations between the components of a system and in verifying how they affect the system from uncertainty's point of view. Firstly, we choose the inputs (i.e. Citizenry, Delegates and Justice) and the output (i.e. E-democracy). Not only the fuzzy sets define inputs, but also the output is a fuzzy set, and all of them consist of multiple fuzzy subsets. Based on human perception, we represent the output as a fuzzy set, because of impossibility of constructing a linear function from the inputs. Thus, our system is subject to a Mamdani fuzzy inference system (MFIS), which is more vague and uncertain, but close to the human perception. The procedure that defines the construction of the inputs and the output has the following steps: *i*) calculate the value of uncertainty (we will presently discuss a proposal), *ii*) create crisp subsets for inputs and output and *iii*) extend the crisp subsets from step *ii* with the value of uncertainty from step *i*.

In order to define the membership functions of the inputs and the output we start from establishing the basic of Justice, the most important pillar of democracy. Taking into consideration the research [31] that stands behind the 2012 World Project of Justice's Rule of Law Index (RLI), we calculate a vector AI which elements are average indices (AI^k) for each country k of the 66 investigated in 2011. RLI is grouped around nine factors: Limited Government Powers, Absence of Corruption, Order and Security, Fundamental Rights, Open Government, Effective Regulatory Enforcement, Access to Civil Justice, Effective Criminal Justice and Informal Justice. Each of these factors has several sub-factors, at least three. Exemplifying, Absence of Corruption relies on: *i*) Government officials in the executive branch do not use public office for private gain; *ii*) Government officials in the judicial branch do not use public office for private gain and *iii*) Government officials in the police and the military do not use public office for private gain. Each of the sub-factors is calculated by a given formula and the result is a value between 0 and 1 [32]. This 0-1 scale also gives us the perfect pretext to use it for the whole model, as Justice is the watcher of E-democracy (see Figure 1) and the other inputs and the output are built relatively to Justice (more on the remaining of this section). In order to obtain the value of a factor is sufficient to determine the average of its sub-factors. In our research, RLI becomes a two-dimensional vector of factors k (RLI^k) and each RLI^k consists of a vector (V^k) of nine elements, but the last and also the informal one is not part of the quantifying process. Thus, RLI consists of only eight formal dimensions and using each element j of vector V^k, an index AI^k is determined for each country k, see (1).

$$AI^k = \frac{1}{8} \sum_{j=1}^{8} V_j^k \qquad (1)$$

Table 2 presents the minimum (a), average (b) and maximum (c) values for the elements AI^k of the vector AI, and the absolute distances from average (d_1, d_2):

Table 2. Minimum, average, maximum and absolute deviations from average for AI

Minimum	Average	Maximum
$a = \min(AI) = 0.33$	$b = \text{average}(AI) = 0.59$	$c = \max(AI) = 0.88$
$d_1 = b - a = 0.26$	0	$d_2 = c - b = 0.29$

Based on adjusted d_1 and d_2, a, b and c we build the membership functions that define the fuzzy subsets of Justice (FJ), as illustrated in Figure 4.

Table 3 presents the parameters of FJ: the values of MF's parameters (VMF), the type of each MF (TMF) and the name of the respective fuzzy subset (NMF), as long with the values derived (VD) from a, b, c, d_1 and d_2. VD has five components: volatility (σ), lower bound (lb) and upper bound (ub) that delimit the crisp interval of each MF and the approximate left value (lv) and approximate right value (rv) of each MF support [33].

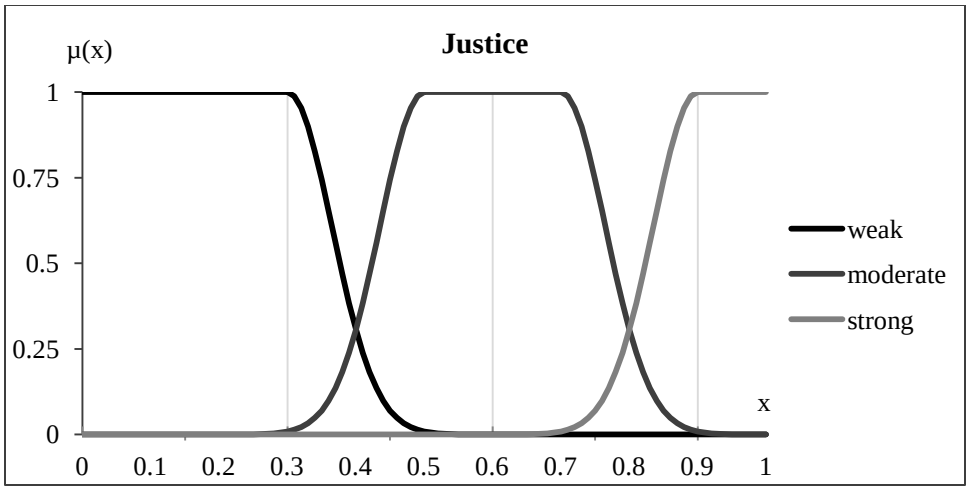

Figure 4. FJ

Comparing Table 2 and Table 3 and regarding Figure 4, we see that the value 0.33 of *a* derived in the value 0.3 of *weak ub* and *moderate lv*. The value 0.59 of *b* became the value 0.6, which is the center of the crisp interval of *moderate*, delimited by *moderate lb* and *moderate ub* (i.e. between 0.5 and 0.7). The value 0.9 represents the *moderate rv* and the *strong lb*; it is derived from value 0.88 of *c*. Not only that these derived values assure a simplified and uniform distribution for MFs of FJ, but it provides a constant value 0.065 for *σ*. Function *gauss2mf* is a standard provided by Matlab environment [34].

Table 3. Values of FJ's parameters

NMF	TMF	VMF	VD				
			σ	lv	lb	ub	rv
weak	gauss2mf	(0.065 0.000 0.065 0.300)	0.065	0.000	0.000	0.300	0.500
moderate	gauss2mf	(0.065 0.500 0.065 0.700)	0.065	0.300	0.500	0.700	0.900
strong	gauss2mf	(0.065 0.900 0.065 1.000)	0.065	0.700	0.900	1.000	1.000

A discussion on the importance of *σ* is necessary at this moment. We have already designated *σ* as volatility, and this is for a strong reason. Volatility describes a variation of an asset price in finance, and is historically calculated using statistical standard deviation over a time series. Volatility is a component that helps establishing a risk-neutral field of interest [35], and it has been successfully applying in finance [36, 37] for the last decades. Instead of using probability fields and volatilities, fuzzy sets theory use human perception and uncertainty [38]. In this paper, we propose a simplified type of volatility, empirically discovered, not a result of a Brownian motion. Volatility *σ* represents the uncertainty on a neutral human perception, and not a risk-neutral measure and it is calculated as an implied value. Implied volatility is already a common topic in finances and especially in option pricing [35, 39].

Taking into consideration the results of Table 2 and Table 3, we have empirically discovered the value 0.065 for *σ* when defining FJ. We practically assume that FJ have crisp values for its MFs, and the fuzzy values are symmetrical extensions, on horizontal axis, of *lb* and *ub* using *σ*, on left and, respectively, right sides of them.

The value 0.065 of *σ* is important because it is not a result of a qualitative process based on a simple human perception, but it emerged from a stationary series of data, which, in author's opinion, gives more relevance to its neutrality. Thus, *σ* is a measure of neutral uncertainty and it helps defining the fuzzy sets based on crisp values. We will use *σ* not only to determine the MFs of FJ, but to determine the fuzzy sets for all the other inputs and for the output, too. As shown in Figure 1, Justice is the watcher of E-democracy and all the other components bind to Justice. So that we will consider *σ* not only the measure

of uncertainty for FJ, but also for the other fuzzy sets of our model of E-democracy, taking into consideration the fact that it has a neutral quantitative worth.

Another option to calculate uncertainty about a general human issue is to conduct some statistical and sociological investigation, such as polls, but they may come upon some drawbacks from at least two perspectives: resources and trust. We can overpass both resources and trust with an index such as RLI. This index includes not declarative assumptions from institutions of democracy but real facts of society, based on some reliable model. This is the reason we believe RLI is an objective expression of democracy's institutions, such as citizenry, justice, government, parliament etc. Fuzzy logic relies on uncertainty, but we do not know if all subjective opinions may be part of this uncertainty, regarding these opinions as outliers. Uncertainty must be build with objective instruments, under the constraint of human common sense.

We also assumed the importance of justice in democracy (see Figure 1) so that we believe the construction of uncertainty should come from manifestations of justice. Democracy is the space of inclusion with an amount of uncertainty and the issue of outliers is not easy to handle [40]. We can avoid studies on beliefs concerning the degree of participation, which, by the way, are very demanding [41], by taken volatility extracted from RLI as a proxy for this uncertainty. Of course, it seems an easy way to cheat on our problem of collecting and analyzing large amount of data, but we truly state that the reflection of human beliefs resides on the way they have structured and organized their society [19]. Thus, RLI gives a global (and national) indication if society's components support democracy as a way of living. RLI also offers a larger perspective than any other method of investigation would bring. These are reasons to extrapolate variables discovered with RLI to the other institutions of democracy, which behave under the watch of justice.

We need also to discuss how we discovered value of σ, which is not a standard deviation of AI, see (1). We may say uncertainty σ is an implied uncertainty, because it verifies in a good manner construction of MFs that define FJ. Initially, we discovered $a = 0.33$, $b = 0.59$ and $c = 0.88$ (see Table 2) as min(AI), average(AI) and, respectively, max(AI). We adjust a, b and c to new values $a^* = 0.3$, $b^* = 0.6$ and $c^* = 0.9$ and decided to build levels of Justice based on these adjusted parameters. We consider that any level under 0.3 is unacceptable for a democratic society, as RLI investigates not only country that have a democratic tradition and institutions according to it, but also not well developed country from a democratic perspective. Participative and deliberative democracy, which finds an expression as E-democracy, is supposed to have higher goals than representative democracies and this is the reason the best score of RLI (i.e. c^*) is only the point where strong Justice would commence. As for moderate Justice, we choose not only one solely value b^* to define its high level (i.e. crisp value) but an interval around b^* where justice can be considered 100% moderate. The interval around b^* is (b^*–0.1; b^*+0.1) and because b^* is the median of a^* and c^* we built MFs of FJ with symmetric configuration (see Figure 4). The values that verifies a Gaussian distribution that has an interval as a peak (i.e. *gauss2mf*) are represented by a^*, b^*-0.1, b^*+0.1 and c^*, along with $\sigma = 0.065$. Thus, MFs of FJ are built with a generalized bell-shaped Gaussian functions using only one constant value for σ, instead of two distinct values, and different values for lb and ub of each MF which are given by a^*, b^*-0.1, b^*+0.1 and c^*, i.e. (0.3;0.5;0.7;0.9), see Table 3. While a^*, b^* and c^* are extracted from AI, value of σ is empirically discovered being that value that verifies MFs of FJ (Figure 4 and Table 3). We may say that the model of E-democracy uses an implied uncertainty σ, determined as a solution of a virtual system of non-linear equations that configure FJ.

It is true that our model is a simplified one, because using only one value for σ constructs a symmetrical bell-shaped Gaussian function which normally requires two values for σ. Relaxing the problem means that we take into account a, b and c or a^*, b^* and c^* along with d_1 and d_2. Still, the

distances between average and extremes are not very different and using two types of uncertainty σ to the left (σ-left) and to the right (σ-right) raises another issue. This means, for a fuzzy subset S of one random universe of discussion U, that different σ-left from σ-right implies that fuzzy values of S do not have the same distribution on both sides of crisp values of S, which also defines S with a non-uniform distribution.

Let us take a didactic example to better understanding the importance of σ-left and σ-right. Taking human age as the universe of discussion from the simplest perspective, and we want to define youth and its range with respect to years lived from beginning to any given time. Figure 5 illustrates a non-uniform distribution for a fuzzy subset called *Youth* (i.e. S), which is part of a fuzzy set *Age* (i.e. U).

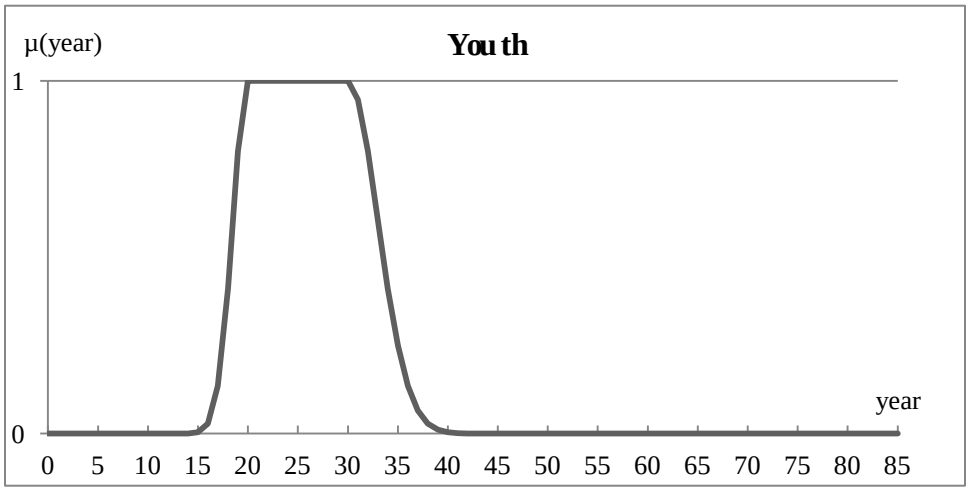

Figure 5. Distribution of Youth

The VMF of the bell-shaped function that define *Youth* are $lv \approx 15$, $ub = 20$, $lb = 30$, $rv \approx 40$, σ-left = 0.33 and σ-right = 0.66. We do not aim at a strict definition of the range of youth, but there are more perspectives: school, experience, work, other's age, politics and even self-perspective. One can be old from school's perspective but crude from a political view and so on. Still, a distribution as Figure 5 displays it is hardly inappropriate from a multi-valence perspective. More, it is quite usual that a person aged 75 to think that a 50 years old person is still young, at least to be retired or to die. In some other cultures there is a true belief that a boy from around 10 years of age is *man* enough to work side by side with his parents. These outliers can raise serious questions about the level of uncertainty and/or about their removal from data that should become more relevant [42].

Taking into consideration this uncertainty about the level of uncertainty it is better to find a proxy that would indicate a value for σ. Democracy is still young, with a certain uncertainty, and the road started a few centuries ago through liberal democracy is only the beginning. More quantitative and qualitative analysis will provide us indicators for institutions of democracy, but until then, we find RLI as a trusted source to extract our proxy for uncertainty.

After discussing the importance of uncertainty we will empirically prove its role in E-democracy, see section 5. All future approaches will take in consideration that σ-left and σ-right are equal, but we believe that a different research should evaluate a relaxation of the problem concerning σ.

3.1 Fuzzy sets of E-democracy

We have already established FJ and MFs of FJ, and we have already determined the value 0.065 for the measure of uncertainty σ. Figure 6 illustrates the fuzzy subsets for Delegates (FD), as long with their MFs.

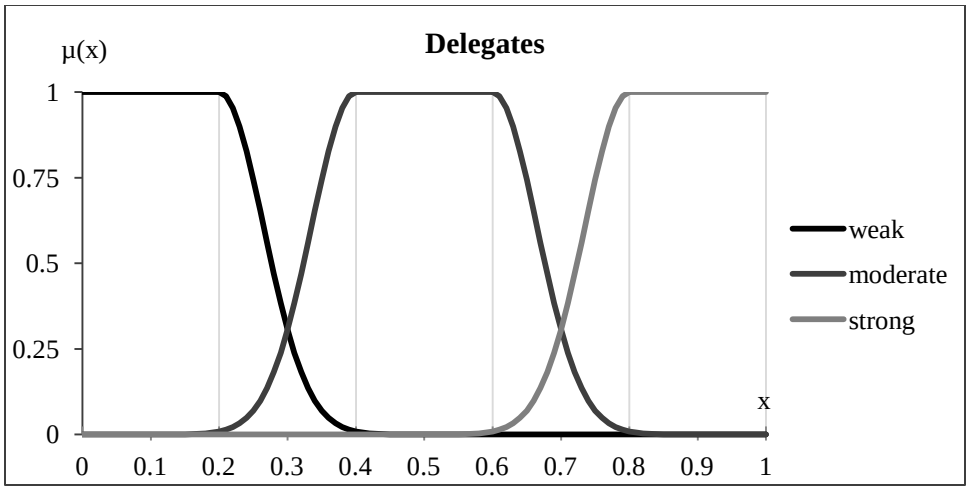

Figure 6. FD

Table 4 presents the parameters values for MFs of FD.

Table 4. Values of FD's parameters

NMF	TMF	VMF	VD				
			σ	lv	lb	ub	rv
weak	gauss2mf	(1.000 0.000 0.065 0.200)	0.065	0.000	0.000	0.200	0.400
moderate	gauss2mf	(0.065 0.400 0.065 0.600)	0.065	0.200	0.400	0.600	0.800
strong	gauss2mf	(0.065 0.800 1.000 1.000)	0.065	0.600	0.800	1.000	1.000

Although the names, types and *σ* of MFs are the same in FD as in FJ, some details differentiate Justice and Delegates in our model of E-democracy. This is because the strictness of evaluating justice is not the same with that of evaluating representatives and the choice for diversified MFs and VD of the inputs relies on author's perception. Still, the values of *lv*, *lb*, *ub* and *rv* are not very important, because we can regard the output relatively to VD for each input. Section 5 of this paper describes this relativity, when different VD for inputs yield correlated values for outputs.

Figure 7 illustrates the fuzzy subsets of Citizenry (FC).

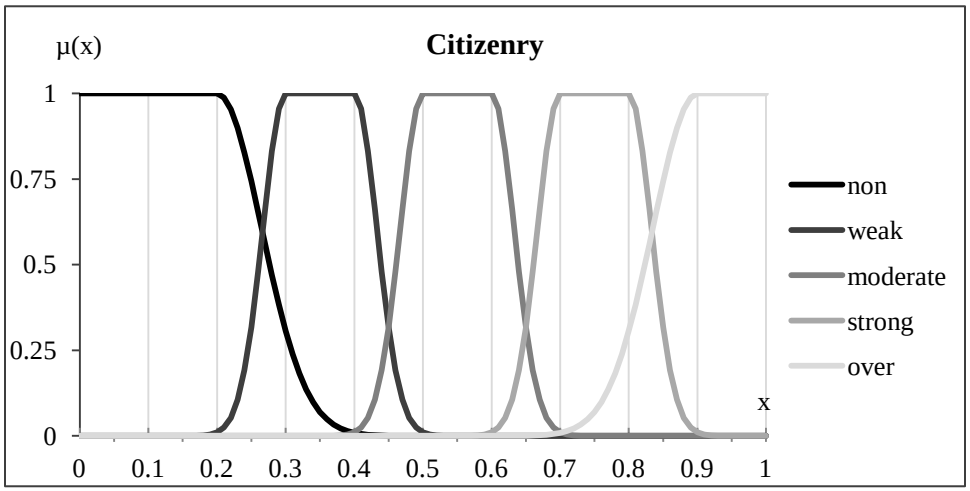

Figure 7. FC

Citizenry is the second most important valor of democracy, almost as important as justice is. The input Citizenry describes the level of participation of the population and not a number of citizens involved in political process. Subsection 3.2 will describe the rules of E-democracy (REDs) and we will

present more details to outline the choice for five MFs of FC. It is important to explain, for the moment, the two values of uncertainty σ used to define FC.

For extreme subsets that define FC, MFs have σ = 0.065, as, in author's opinion, the uncertainty for FC *non* and FC *over* is at a high level. The reason for this affirmation is that *non* represents non-participation, a total disinterest from citizens for political process, which is a peril for democracy. *Over* describes over-reaction of citizens, a desire to substitute the other institutions of democracy to people's power, which is better known as tyranny of majority from ancient [18] to modern times [3, 43].

Table 5. Values of FC's parameters

NMF	TMF	VMF	VD				
			σ	lv	lb	ub	rv
non	gauss2mf	(1.000 0.000 0.065 0.200)	0.065	0.000	0.000	0.200	0.400
weak	gauss2mf	(0.325 0.300 0.325 0.400)	0.325	0.200	0.300	0.400	0.500
moderate	gauss2mf	(0.325 0.500 0.325 0.600)	0.325	0.400	0.500	0.600	0.700
strong	gauss2mf	(0.325 0.700 0.325 0.800)	0.325	0.600	0.700	0.800	0.900
over	gauss2mf	(0.065 0.900 1.000 1.000)	0.065	0.700	0.900	1.000	1.000

The other three subsets of Citizenry: *weak*, *moderate* and *strong* are components of what a virtual single subset (VSS) might have been. Accordingly to FJ and FC, VSS would have had the same relative values for VD, with σ = 0.065. However, a more precisely characterization of FC is required and, thus, the FC *weak*, *moderate* and *strong*, using half of the value of regular uncertainty, are bounded by the FC *non* and *over*, see also Table 5. This is a consequence of a smaller interval of values that define the MFs for the three subsets, which also means that precision is higher and the uncertainty is lower, i.e. σ = 0.0325. We will give more explanations for this configuration of FC in subsection 3.2 when REDs are defined.

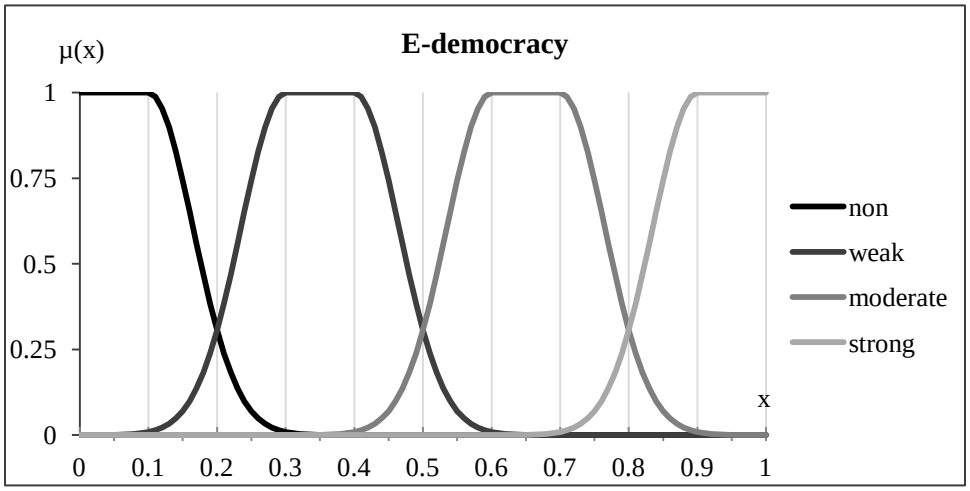

Figure 8. FE

Figure 8 illustrates the fuzzy subsets of the output E-democracy (FE).

Table 6. Values of FE's parameters

NMF	TMF	VMF	VD				
			σ	lv	lb	ub	rv
non	gauss2mf	(1.000 0.000 0.065 0.100)	0.065	0.000	0.000	0.100	0.300
weak	gauss2mf	(0.065 0.300 0.065 0.400)	0.065	0.100	0.300	0.400	0.600
moderate	gauss2mf	(0.065 0.600 0.065 0.700)	0.065	0.400	0.600	0.700	0.900
strong	gauss2mf	(0.065 0.900 0.065 0.800)	0.065	0.700	0.900	1.000	1.000

The output has four subsets, but the values of output VD are relative to the first two inputs described: Justice and Delegates. This means that σ = 0.065 for each MF and, thus, the level of uncertainty is the same for all subsets of FE. Table 6 presents the values of parameters of FE.

Before we discuss the fuzzy rules of our model, let us recapitulate the assumptions made to design the fuzzy sets of the inputs and the output. Firstly, we have introduced σ, a measure for uncertainty, which empirically emerged when having build FJ, based on RLI and AI. Secondly, we have diminished σ to almost half of its value, when more precisely crisp intervals define subsets of FC and, accordingly, the level of uncertainty is lower. Each time uncertainty changes its value we use the same approach of building FC, see subsection 5.3. Thirdly, we have developed a system based on a relative 0-1 scale for each input and for the output, too (we will discuss the relativity of our E-democracy model in section 5). Lastly, we used Matlab *gauss2mf* [34] to build the MFs for inputs and output. Consequently, we will have a Matlab approach when describing an optimization algorithm that finds the minimum and maximum of E-democracy's output, as well as the inputs that yield these results, see section 4.

3.2 Fuzzy rules of E-democracy's model (REDs)

There are eight REDs defined in this paper, and each of them has a logical and political support.

RED 1) If (Citizenry is *non*) or (Justice is *weak*) then (E-Democracy is *non*)

A lack of participation from the citizens is only a totalitarian or, at most, an authoritarian society [18]. If justice has a low level, the rules and regulations of society have a poor foundation and the system is predisposed to subjectivism, which leads, once again, to totalitarianism or authoritarianism [3, 20]. At least one of the conditions must be satisfied, either Citizenry is *non* or Justice is *weak*, to yield a *non* E-democracy. This is the only rule of the REDs that have OR as connector.

RED 2) If (Citizenry is *weak*) and (Justice is not *weak*) then (E-Democracy is *weak*)

We have already established that weak justice leads to non-democracy. If we have anything else for justice but *weak*, and we have a *weak* level for citizenry, which means a feeble implication from citizens, E-democracy is *weak* [23]. Under the watching of justice, members of society must participate to political process. Voting only is herein a weak form of participation from citizens that should also get the equiprobable chances to access the public offices [5].

RED 3) If (Citizenry is not *non*) and (Justice is *moderate*) then (E-Democracy is *weak*)

We already defined citizenry's non-participation as a sufficient condition to a non-democratic society. If we exclude the FC *non* and we have one of the FC: *weak*, *moderate*, *strong* or *over* and we have FJ *moderate*, we only get a FE *weak*. In other words, moderate justice leads to a poor level of democracy. This seems logical and historically proved, because subdued justice becomes an instrument of different groups or private interests [23] and a moderate justice is just a form of injustice [7].

RED 4) If (Citizenry is *not non*) and (Justice is not *weak*) and (Delegates is *not moderate*) then (E-Democracy is *weak*)

If we exclude non-participation and unfair justice, each of them already described as sufficient for *non*- or *weak*-democracy, we consider *weak* or *strong* representatives as undesirable for an optimum level of democracy. Strong representatives lead to modern political clientelism [44], or, even worse, to an oligarchic system. A weak level for representatives is yet not desirable, as eliminating the liberal legislative and executive powers implies either tyranny of majority or poor functionality in state's administration [18].

The first four REDs have a more or less a general character, but the next rules are very specific, providing unique logical combinations of inputs.

RED 5) If (Citizenry is *moderate*) and (Justice is *strong*) and (Delegates is *moderate*) then (E-Democracy is *moderate*)

RED 6) If (Citizenry is *strong*) and (Justice is *strong*) and (Delegates is *moderate*) then (E-Democracy is *strong*)

Moderate involvement from representatives and a high level of justice are conditions for an acceptable level of democracy. It is citizenry's participation that indicates the level of E-democracy; if it is *moderate* or *strong*, so is the level of democracy, *moderate* or, respectively, *strong*. A moderate participation is, in author's opinion, the involvement in electoral process, as candidates or supporters of candidates. A strong participation is a continuous engagement of citizens in political process, using digital instruments, when possible, to supervise the elected representatives and to propose solutions for state and community matters.

RED 7) If (Citizenry is *over*) and (Justice is *not weak*) and (Delegates is *moderate*) then (E-Democracy is *weak*)

We have already established that representatives must have a moderate implication in order to obtain a level for democracy that is not weak. Weakness of justice means non-democracy and average level of justice already yields poor democracy. If citizenry has an *over*-participation, which means that either majority decides in all matters or the only concern of citizens is political decision and negligence of other fields of human life, democracy reaches a *weak* outcome. Even if justice is strong, it might become an instrument in majority's hands and moderate activity from representatives cannot balance citizen's over-reaction or state's administration is difficult to be achieved.

RED 8) If (Citizenry is *over*) and (Justice is *strong*) and (Delegates is *strong*) then (E-Democracy is *moderate*)

This rule is an exception from RED 4 and RED 7, when citizenry's over-participation and a level that is not moderate for delegates are each sufficient to determine a weak level for E-democracy. If all inputs are at their highest standard it could be intuitively enough to have the best outcome for democracy. The principle of "checks-and-balances" might be invoked, but we believe that, on medium and long term, this would lead to a corrosion of all three institutions. This combination probably gives, on short-term, a strong democracy, but E-democracy is a long- and very long-term commitment. Thus, we are convinced that the *highest* levels of inputs yield, at most, a *moderate* E-democracy.

Before we continue with the mathematical formalization of REDs, the three inputs require a short discussion, to better understanding the author's position when defining the rules of E-democracy. Citizenry is not just a body that includes all of the citizens, but it is an expression of general will (*volonté générale*) and common interest [23], without neglecting the freedom of individuals [6, 45]. This will is an expression of citizenship [46], community spirit [47] and the urge for deliberation, beyond the prejudices of ideologies [7, 24]. Justice is the institution that prevents citizens or representatives from imposing their tyranny [3, 4, 43]. It is also the watcher and preserver of the rights of each community, minority or individuals and is the key for a society of inclusion, no matter of religion, gender, race, age, beliefs etc. [5]. The novelty that E-democracy brings with it is the information and communication technologies (ICT). What once was so difficult to facilitate in terms of participation and discussion, communication and socialization, informing and abstract simulation it is now achievable by ICT. Is there a way that ICT could improve the juridical system? The answer is definitely yes, and there will be a time when E-justice is a common term. There have been, for the last decades, attempts to emphasize the role of ICT in judicial system [48], the need for E-justice [49] and the importance of the management of juridical system [50]. Let us not forget the portals dedicated to justice and the simple sites that provide legal advice and guidance. Nonetheless, these instruments of E-justice are important, but what we need is even more: facilitate the deliberation over different typologies of juridical principles and, in a not so long future, data mining and semantic analysis on law texts and meaning. A system of justice that is not prepared to reinvent itself is not a strong one [7], and we should expect ICT to make easier the transition

to a transnational justice [51], in a global knowledge society. To conclude our final word on principles that governs E-democracy, we outline the representative's framework and we state that all types of delegates are responsible in front of the stakeholders: citizens. The parliament is a legislative body and, compared to an economic organization, its members are comparable to the leaders, the assembly/board that defines the vision and mission of the state. In the same analogy, the government is the managerial board that, together with the parliament, establishes the objectives of the state, but only the government pursues the task of fulfilling these objectives, under the supervision of the parliament. The president or the monarch is a symbol of society's organization, not seen as a general manager, but more as a public relations representative. Even if president might have increased powers in a semi-presidential or presidential regime, they represent a democratic institution as long as they do not act like tyrants or interfere in parliament or government attributions.

Participation in E-democracy is the key for knowledge society that acts, with the help of representatives and justice, for the benefit of individuals and community. However, the participation cannot be the same on all levels of society and we propose a model of citizens' involvement in Figure 9, based on geographical constraints.

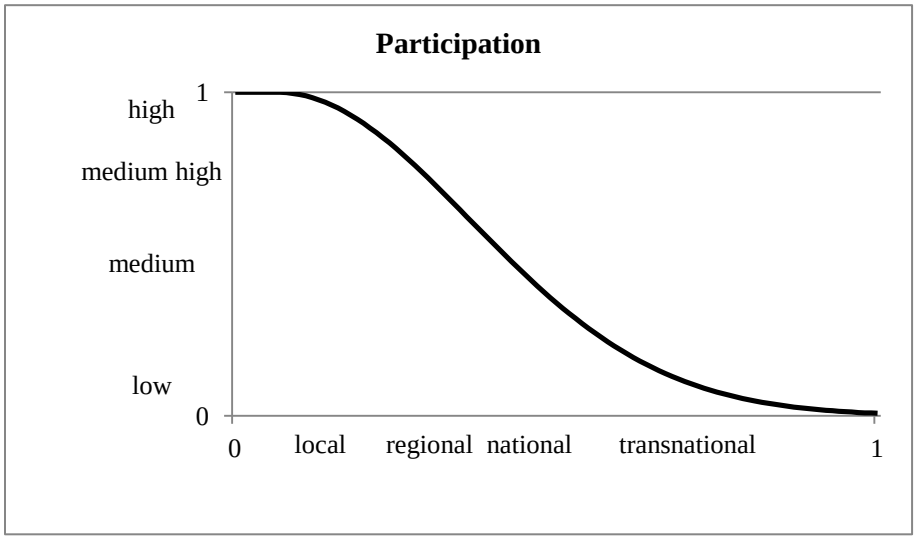

Figure 9. Different levels of participation

Taking into account REDs and Figure 9, we believe that E-democracy should rely on both considerations: the need for participation and deliberation on one hand, and the geographical constraints on the other hand. We do not discuss any detail in this paper about levels of participation subject to geographical constraints, but we only point out the constraints that we should take into consideration when developing an architecture of E-democracy's model.

3.3 Fuzzy logics of E-democracy's model

Before we continue with discussions and interpretations of E-democracy's outputs we remind the components (cMFIS) of MFIS [52].

cMFIS 1) Fuzzification - building the fuzzy subsets of the inputs, defined by MFs, using crisp input values (see Figures 4, 6 and 7 and Tables 3, 4 and 5).

cMFIS 2) Definition of knowledge base - applying fuzzy operators by building rules of MFIS (see REDs, subsection 3.2).

cMFIS 3) Implication method - determining an intermediary output (O) for each rule, based on a single or multiple fuzzy subsets (see Figure 8 and Table 6).

cMFIS 4) Aggregation of all outputs - combining all intermediary outputs of type *O*, from each rule, to create a single final output (*F*).

cMFIS 5) Defuzzification - extracting the crisp value from final output *F*.

The only part of cMFIS that remains unchanged is the knowledge base; all the other components dynamically change with every single change of one input crisp value. Figure 10 illustrates interrelation within cMFIS.

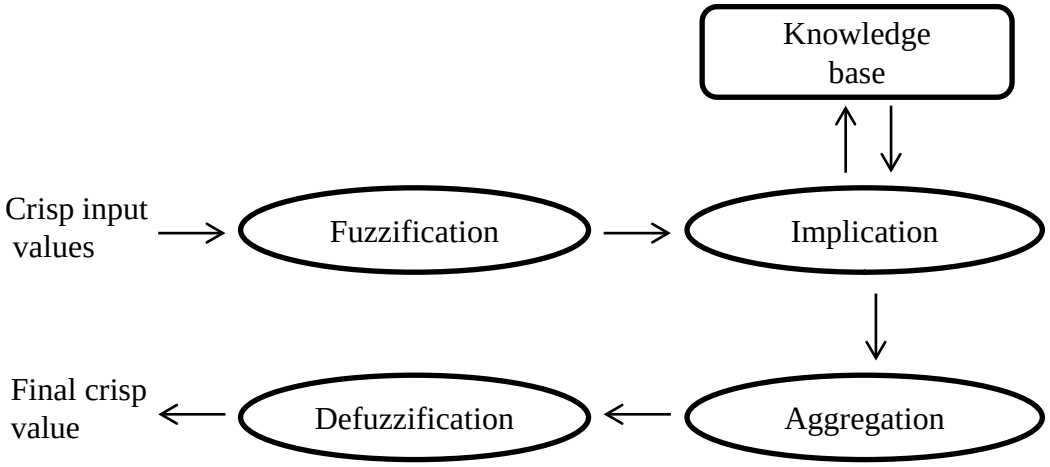

Figure 10. Components of MFIS

All five components have mathematical formalizations and for knowledge base it will be minutely presented such formalization, see subsection 4.1. We discuss now only the mathematical functions used for the other four components.

- Fuzzification: function *min* (minimum) is used for operator AND, function *max* (maximum) corresponds to operator OR. These functions are very common for logical operators and further discussions on this matter are irrelevant.
- Implication: *min* or *prod* is the alternative. The first one, coined by Mamdani, truncates the output fuzzy subset while the second, belonging to Larsen, scales the output fuzzy subset [52]. Formula (2) describes the mathematical formalization of implication, for *n* inputs and an output on position *n*+1.

$$\mu^{*}_{n+1}(x) = \alpha \times \mu_{n+1}(x) = prod(\alpha, \mu_{n+1}(x)) \text{ - Larsen}$$
$$\mu^{*}_{n+1}(x) = \alpha \wedge \mu_{n+1}(x) = \min(\alpha, \mu_{n+1}(x)) \text{ - Mamdani}$$
$$\text{where } \alpha = \underset{j=1}{\overset{n}{\Delta}}(\mu_j),\ \Delta \text{ is logic operator (i.e. AND/OR),} \qquad (2)$$
$$\text{and } \mu^{*}_{n+1} \text{ is the final value of the output } F,$$
$$\mu_{n+1} \text{ is the value of intermediary output } O,$$
$$\mu_j \text{ is the value of input } j$$

In a didactic example in Figure 11a α limits but also multiplies the values of output MF on vertical axis, while on Figure 11b α only limits the values of output MF.

- Aggregation: this time the alternative is given by three possibilities, with specific functions: max, probor (probabilistic OR) and sum. While max and sum are ubiquitous, we present the mathematical formalization for the third one in formula (3).

$$probor(a,b) = a + b - a \times b \qquad (3)$$

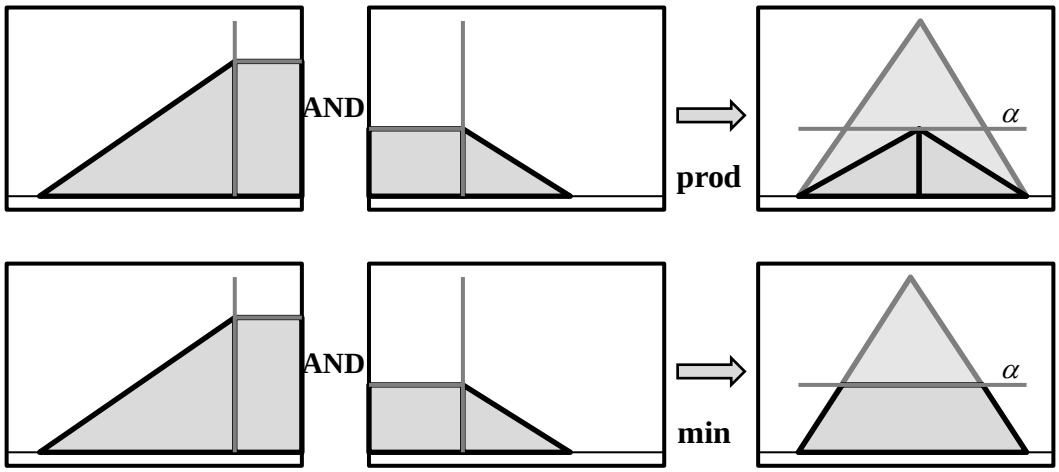

Figure 11. Fuzzy implication methods: a) prod; b) min

- Defuzzification: there are five possibilities to obtain the crisp value of the final output *F*. Three of the functions that determine the result extract the lower, the medium and, respectively, the largest of maximum of the aggregated output set *F*. We do not want a result that relies only on high outcomes, as this may be associated, from a human perspective, with elitism. In our model, there is not the case to be afraid of low convexity and extreme values [52], and, thus, to choose one of these methods based on maximum. On the contrary, if we must find a way to a widely accepted point, a most representative value has to be the solution of our optimum output. For the other two methods to choose from in our model of E-democracy: bisector method divides *F* in two equal areas, while centroid yields the x-value for the point that is the center of gravity for *F*. We believe that the point that balance the output is more suitable for E-democracy than the point that gives two views equal from the perspective of space, but not necessary from the perspective of importance. Thus, we choose the centroid method and we present the formalization for discrete values as, anyhow, values on continuous space are not explicitly subject to computational approach.

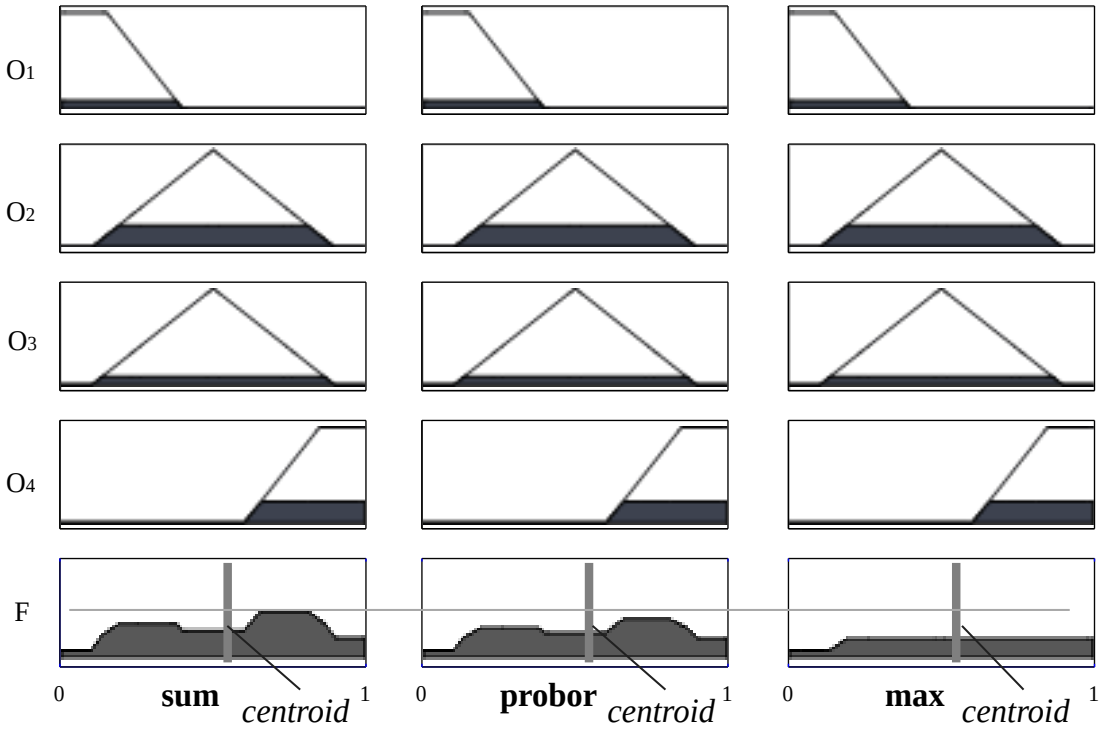

Figure 12. Aggregation with sum, probor and max; defuzzification with centroid

Formula (4) presents the centroid method for a number K of discrete values used to generate MF.

$$q^* = centroid(F) = \frac{\sum_{k=1}^{K} \mu(x_k) \times x_k}{\sum_{k=1}^{K} \mu(x_k)} \quad (4)$$

Figure 12 illustrates, for a theoretical MFIS with four rules, two inputs and an output, that centroid method yields similar results for the three cases of aggregation. More, this is not a consequence of similar final fuzzy set *F*; in case of *max* aggregation, the shape of *F* is very different. For *probor* and *sum* aggregation, the shapes of *F* are similar, but in different proportions, given by the fact that *probor* is a particular case of *sum*, see formula (3).

We will discuss the relativity of E-democracy to methods of cMFIS in section 5 after presenting AOE in the next section, thus providing support to calculating different optimum outputs.

4. Algorithm that finds an optimum E-democracy's output (AOE)

Appendix I presents AOE in detail and Appendix II provides a Matlab implementation of it. We only describe here the important parts of AOE and there are three important components of AOE: algorithm that generates antecedents and consequents (AGAC), algorithm that finds an initial solution (AIS) and AOE itself. The mathematical and logical formalization of AOE emerge from a Matlab approach.

4.1 AGAC

AGAC consists of six steps and they are briefly described without any other mathematical formalizations (see Appendix I for details).

AGAC 1) Let us take *y*, a vector that contains the maximum possible values of each input and output (i.e. the number of MFs of each input and output). Let us take *Y* a matrix of sets obtained from generating integer positive values from 1 to each element of *y*, for each column of *Y*. *Y* contains as many rows as REDs and it is a three-dimensional vector / list.

AGAC 2) Let us take *P* a matrix (bi-dimensional vector) that contains only the rows that have at least one negative value for inputs and output.

AGAC 3) Let us take *R*, a matrix that contains sets of positive values (thus, *R* becomes a three-dimensional vector). *R* evolves through removing positive elements, with negative signs in *P*, from *Y* or by keeping the positive values of *Y*, which also have a positive sign in *P*.

AGAC 4) Let us define *r*, a vector that contains the number of rows of intermediary matrix *M** of the final result matrix *M*.

AGAC 5) Let us calculate *r*, with a number of elements equal to rows of *P* and let us build each intermediary matrix *M**, obtained from all combinations of positive integer values of *R*.

AGAC 6) Let us concatenate each intermediary matrix *M** with the final matrix *M*.

Final matrix *M* contains mathematical formalized REDs with positive inputs and outputs (REDPIs) and it will act as an input to obtain initial solutions for AOE.

4.2 AIS

The purpose of AIS is to automatically offer an initial solution, stored in a vector *x* and an initial scalar result *q* yielded by this vector *x*.

AIS 1) Let us take M obtained with AGAC and let us read t the positive integer value that defines the fuzzy subset of the output which is the target for AOE.

AIS 2) Let us extract from matrix M a matrix G that contains value t on the column of consequents.

AIS 3) Let us calculate a matrix X that contains as many rows as matrix G. Elements of X are determined using centroid method or the median of lb and ub for each element of G that describes a certain fuzzy subset of inputs; see formulae (4) or, respectively, (6). Matrix X is a bi-dimensional vector that contains, on its rows, several vectors of type x.

AIS 4) Let us determine the value of q such as q is the result of an MFIS applied to vector x (i.e. row of X). Let us define the function mfis that takes x as parameter and yields q.

$$q = mfis(x) \tag{5}$$

4.3 AOE

We also briefly present steps of AOE, see Appendix I for details, and we provide a Matlab implementation of AOE in Appendix II.

AOE 1) Let us define the type (maximization or minimization), precision and bounds for AOE.

AOE 2) Let us read or calculate, using AIS, vector of initial inputs x and output q.

AOE 3) Let us calculate auxiliary variables for numerically differentiability of inputs.

AOE 4) Let us calculate, through unknown number of iterations, vector of final input solution x^* and final output solution q^*, based on auxiliary variables calculated in AOE 3.

We already presented, see Appendix I, an improvement of AOE and some proof of its accuracy for a process of minimization in a time series.

AOE and its components somehow follows the steps of BPE (see Table 1) only that it does not use any activation function or derivative of a function, but it is a direct search (which is a human approach that transforms this technique into a paradigm that PDI may use). AOE is trial and error method that finds a minimum or a maximum with a slow pace. Algorithms that rely on numerical derivatives have a faster velocity, but they do not always yield a solution (see a comparison for bisection and Newton method for solving a nonlinear equation [39] or comparison of AOE and other methods for solving a system of nonlinear equations in Appendix I).

Next section uses AOE in order to determine minimum or maximum of E-democracy as an output in FME when applying several sensitivity analyses.

5. E-democracy's outputs obtained with AOE

In this section, using AOE, we will prove that some methods of cMFIS hardly influence E-democracy's optimum output. We will also discover some particularities of FME, after choosing the functions that describe the fuzzy logic of FME. For the moment, we discuss the method which determines the value of initial vector x. One of the methods is the centroid, presented in formula (4), which this time deal with each of the input MF that is part of any REDPI that determine the target output t.

$$x_i = \begin{cases} (\mu_i^{bs} + \mu_i^{bd})/2, \text{ if input } i \text{ is part of RED (i.e. it is not zero in REDPI)} \\ (l_i^0 + l_i^1)/2 \ (i.e.\ 0.5), \text{ other (input } i \text{ is zero in REDPI)} \end{cases} \tag{6}$$

where μ_i^{bs} and μ_i^{bd} are the lower and, respectively, the upper bound of any MF of input i

and $[l_i^0; l_i^1]$ is the range of MF of input i; $i = 1, n (n = 3)$

The other method to determine each element of vector x is presented in formula (6), and it takes into consideration each input's MF that is part of any REDPI determining the target output t.

AOE calculates all the results for x^* and q^* with a precision of 0.0001, when some other precision is not mentioned.

5.1 Relativity of E-democracy to methods of MFIS

Any MFIS depends on the methods used for its components cMFIS, i.e. logical-mathematical functions. We have already discussed the fuzzy logic of E-democracy in subsection 3.3 and now we present results using different methods for cMFIS.

Table 7 reveals the scores of maximization of E-democracy with AOE, using the following constant functions for cMFIS: AND - min, OR - max, defuzzification - centroid. The results use distinct functions of cMFIS for implication and aggregation of E-democracy.

Table 7. Maximization of E-democracy with distinct methods of cMFIS, using (6)

Implication	Aggregation	x	q	x^*	q^*	q/q^*
min	Max	(0.75 0.95 0.5)	0.826707	(0.7010 0.95 0.5)	0.892392	1.0787
min	Probor	(0.75 0.95 0.5)	0.825851	(0.6995 1.00 0.4)	0.887341	1.0735
min	Sum	(0.75 0.95 0.5)	0.825810	(0.6997 1.00 0.4)	0.887173	1.0733
prod	Max	(0.75 0.95 0.5)	0.857016	(0.6986 0.95 0.5)	0.898241	1.0476
prod	Probor	(0.75 0.95 0.5)	0.856620	(0.7000 1.00 0.4)	0.897082	1.0467
prod	Sum	(0.75 0.95 0.5)	0.856597	(0.7001 1.00 0.4)	0.897028	1.0467

The vector x from Table 7 contains three identical values, obtained with formula (6), no matter the distinct methods of cMFIS, which do not affect anyhow the process of generating initial inputs, see AGAC. On the contrary, the values of q, x^* and q^* differ when methods of cMFIS change, proving a sort of dependence. We also present in the last column of Table 7 an indicator of relative change of the output value from the initial to the optimum result.

Before we continue with further investigations we will present in Table 8 results obtained in similar conditions as in Table 7, only this time vector x is not determined by formula (6), but by the method of centroid, applied to each element of x and not to q, see formula (4).

Table 8. Maximization of E-democracy with distinct methods of cMFIS, using (4)

Implication	Aggregation	x	q	x^*	q^*	q/q^*
min	Max	(0.75 0.90688 0.5)	0.826707	(0.7010 0.95 0.5)	0.892392	1.0787
min	Probor	(0.75 0.90688 0.5)	0.819582	(0.6995 1.00 0.4)	0.887341	1.0817
min	Sum	(0.75 0.90688 0.5)	0.819166	(0.6997 1.00 0.4)	0.887173	1.0820
prod	Max	(0.75 0.90688 0.5)	0.857016	(0.6986 0.95 0.5)	0.898241	1.0476
prod	Probor	(0.75 0.90688 0.5)	0.853151	(0.7000 1.00 0.4)	0.897082	1.0510
prod	Sum	(0.75 0.90688 0.5)	0.852920	(0.7001 1.00 0.4)	0.897028	1.0512

Data from Table 7 and Table 8 show some constant dependencies. No matter the initial values of inputs stored in vector x, they changed in the same direction when they do change, with similar proportions leading to similar final optimized input x^* and identical final optimized output q^*. It proves that the initial values of x are important only when using *max*-aggregation, with both *min*-implication and *prod*-implication. This is a consequence of the fact that *max*-aggregation levels down the final output, equalizing the intermediary outputs of type O that lead to a flat final output F, see also Figure 12. For this reason we will exclude this kind of approach, as E-democracy is not a totalitarian system that searches to bring its components to the same level, but to extract the most representative value from diversity, under the watching of well-defined rules. We only refer, for the moment, to *max*-aggregation for the purpose of proving the relativity of E-democracy to methods of cMFIS, but not for relativity to inputs scale, which will be discussed in the next subsection. More, *max*-aggregation is easily satisfied and it does not demand a high participation from all inputs; in our model of E-democracy only Citizenry is

subject to AOE's iterations. And, of course, *max*-aggregation corresponds to a more optimistic point of view, proved also by comparing the results of *q** in both Table 7 and Table 8.

In order to prove the relativity of E-democracy's output to methods of cMFIS, we remind the formalization of Pearson's statistical correlation indicator in formula (7).

$$\rho = \frac{\operatorname{cov}(X,Y)}{\sigma_X \times \sigma_Y} = \frac{\sum_{i=1}^{N}(X_i - \bar{X})^2 \times (Y_i - \bar{Y})^2}{\sqrt{\sum_{i=1}^{N}(X_i - \bar{X})^2} \times \sqrt{\sum_{i=1}^{N}(Y_i - \bar{Y})^2}} \tag{7}$$

where N is the number of observations and ' ¯ ' is operator average

Table 9 presents the relativity of methods of cMFIS to E-democracy, based on data from Table 7 and Table 8, taking into consideration only the *min*- and *prod*-implication as principal indicators.

Table 9. Maximization of E-democracy with distinct formulae for calculating *x*

Implication	Using formula (6)		Using formula (4)	
	ρ(q, q*)	average(q/q*)	ρ(q, q*)	average(q/q*)
min	0.999936	1.07607	0.999785	1.08172
prod	0.999953	1.04751	0.999938	1.05044

Using formula (7), in Table 8 we calculate correlation of *q* and *q** for only three observations and this might not be statistical efficient for predictions. In our case coefficient of correlation *ρ* is only used to empirically verify a linear dependency between an initial and a final output. This is the dependency we look for and not between *x* and *x**, because the key of our research is E-democracy's output as an expression of the optimum inputs. When using formula (6) we have not only an infinitesimal better correlation for both *min*- and *prod*-implication, but we also have, in average and in particular (see Table 7 and Table 8) a smaller relative change from the initial output *q* to the final output *q**. Thus, no matter the type of aggregation and implication, with functions *min* and *max* for logical operators *AND* and, respectively, *OR* and centroid approach for defuzzification, we have an almost perfect linear correlation between *q* and *q** with both formulae (4) and (6). On the other hand, we have better guess solution (i.e. *x* - initial vector solution and *q* - initial output) with (6) than with (4), with a slight advantage for *prod*-implication. Taking into consideration the results obtained with *probor*- and *sum*-aggregation are almost identical (see Table 7 and Table 8) we will choose the latter for the following investigations, because the former does not give us the same larger perspective on final output *F*. We may say that, to keep a balance between implication and aggregation, we select prod- in place for min-implication because the former truncates intermediary outputs of type *O*, while sum- does not proportionally decrease final output *F* as probor-aggregation does, see Figure 11 and Figure 12.

Table 10. Minimization of E-democracy with distinct formulae for calculating *x*

Formula	*x*	*q*	*x**	*q**
(6)	(0.1 0.15 0.5)	0.0931175	(0.1 0.15 0.5)	0.0931175
(4)	(0.14141 0.19059 0.5)	0.0931175	(0.14141 0.19059 0.5)	0.0931175

We have not yet discussed about relativity of the model in case of minimization. Unfortunately, FME cannot allow us to make as many investigations as we did for relativity to methods of cMFIS in case of maximization. Table 10 shows the simplicity of the model when we take into consideration relativity to cMFIS in case of minimization, using the following approaches: AND - min, OR - max, defuzzification - centroid, implication and aggregation - indifferent.

In case of minimization, the results depend only on the initial guess of inputs. Both formulae (4) and (6) lead to the same final output q^*, but with different final inputs x^*, which are actually the initial solution x. The final result is unique but there is infinity of solutions, due to the RED 1, which establishes that either Citizenry or Justice directs to non E-democracy. RED 1 does not take into account Delegates, and it has the value of 0.5 as conventionally assigned, see formula (6).

Table 11 presents some combinations with inputs that yield a non E-democracy minimum or close to minimum.

Table 11. Combination of initial inputs that yield non E-democracy

x	q	x^*	q^*
(0.1600 0.35 0.5)	0.0931175	(0.1 0.50 0.5)	0.0931175
(0.2001 0.30 1.0)	0.0931179	(0.2 0.30 1.0)	0.0931175
(0.7000 0.14 1.0)	0.0931176	(0.14141 0.19059 0.5)	0.0931175

We notice in Table 11 that different combinations of inputs yield the same value for the final input q^*. This is already discussed in Appendix I, when we explained that there are infinity of solutions x^* but only one final solution q^*. Boundaries of final inputs and output of E-democracy will be minutely presented in subsection 5.4. We conclude this subsection by stating that methods of cMFIS do affect the results only in a relative way, through a linear correlation between initial guess and final result, empirically proved with AOE for a precision of 0.0001.

5.2 Relativity of E-democracy to inputs scale

In this section we will try to prove that a linear correlation exists between the parameters of MFs and the optimum output. Thus, if we change with a certain degree all parameters that define MFs of an input, we obtain a similar result. By changing with a certain degree we mean that we increase or decrease by some unit the values of lb and ub for each MF of an input in case of maximization or minimization. Keeping the same 0-1 scale of any input, we only shift all MFs with some unit and preserve the value of σ.

Table 12 presents the impact of alternatively shifting all MFs of any input and keeping both σ and all MFs of the output at the same initial values. The indeces assigned for inputs for the next analyses are the following: Citizenry - 1, Justice - 2 and Delegates - 3.

Table 12. Two types of optimization after shifting all MFs of any input

Type	x	q	x^*	q^*	Unit	Index
min	(0.1 0.15 0.5)	0.0931175	(0.1 0.15 0.5)	0.0931175	0	-
min	(0.05 0.15 0.5)	0.0931175	(0.05 0.15 0.5)	0.0931175	-0.1	1
min	(0.1 0.1 0.5)	0.0931175	(0.1 0.1 0.5)	0.0931175	-0.1	2
min	(0.1 0.15 0.5)	0.0931175	(0.1 0.15 0.5)	0.0931175	-0.1	3
max	(0.75 0.95 0.5)	0.856597	(0.6994 1 0.4)	0.897028	-0.1	-
max	(0.65 0.95 0.5)	0.856597	(0.5994 1 0.4)	0. 897028	-0.1	1
max	(0.75 0.85 0.5)	0.856597	(0.6994 1 0.4)	0. 897028	-0.1	2
max	(0.75 0.95 0.4)	0.856597	(0.6994 1 0.3)	0. 897028	-0.1	3

In case of minimization there are no modifications after shifting the MFs, due to the REDs and configuration of the inputs fuzzy sets (see also Table 9). In case of maximization we observe that the output q^* does not change, and it is identical for all three transformations of MFs. For both minimization and maximization the values of x changes proportionally, except for Delegates in case of minimization,

because its value of 0.5 is conventionally assigned. When maximizing the output we observe that x^* changes proportionally for Citizenry and Delegates, but not for Justice. This in an important observation, if not the most important for FME.

Remark 1: *Justice always has the tendency to reach its maximum value in order to obtain a corresponding maximum output*.

On the contrary, Citizenry and Delegates depend on the values of parameters that define their MFs and they have a scaled impact on E-democracy. Table 13 shows other proves for the importance of Justice, this time with respect to the precision used to find out the maximum result.

Table 13. Justice and maximization of E-democracy, using different precisions; NI - number of iterations

x	*q*	*precision*	*q**
(0.7 0.99 0.4)	0.897	0.01	38
(0.7 0.995 0.4)	0.89701	0.005	78
(0.699 0.999 0.4)	0.897024	0.001	399
(0.6995 1 0.4)	0.897027	0.0005	800
(0.6994 1 0.4)	0.897028	0.0001	4006

Table 14 presents another perspective of relativity of E-democracy to inputs scale. This time we will shift all MFs of each input and we will transform the 0-1 scale in 0-0.9 scale for each shifted input.

Table 14. Optimization after rescaling each input and shifting its MFs

Type	*x*	*q*	*x**	*q**	Unit	Index
min	(0.1 0.15 0.5)	0.0931175	(0.1 0.15 0.5)	0.0931175	0	-
min	(0.05 0.15 0.5)	0.0931175	(0.05 0.15 0.5)	0.0931175	-0.1	1
min	(0.1 0.1 0.5)	0.0931175	(0.1 0.1 0.5)	0.0931175	-0.1	2
min	(0.1 0.15 0.5)	0.0931175	(0.1 0.15 0.5)	0.0931175	-0.1	3
max	(0.75 0.95 0.5)	0.856597	(0.6994 1 0.4)	0.897028	-0.1	-
max	(0.65 0.95 0.5)	0.856597	(0.5994 1 0.4)	0.897028	-0.1	1
max	(0.75 0.85 0.5)	0.856597	(0.6994 0.9 0.4)	0.897028	-0.1	2
max	(0.75 0.95 0.4)	0.856597	(0.6994 1 0.3)	0.897028	-0.1	3

First observation is that, in case of maximization, rescaling and shifting proportionally MFs with new scale lead to perfectly correlated, using formula (7), initial output and inputs with optimum output and, respectively, optimum inputs (i.e. the correlation is 1 between first column vector of an initial input x and first column vector of the final input x^*). In case of minimization it does not affect, anyway, the best results by keeping the same behavior as in previous investigations, see Table 12.

Second observation is that Justice converges to its maximum possible value (i.e. *ub* and/or *rv*). Thus, we have another empirically prove that justice may be the most important institution of democracy with respect to REDs.

Finally, we notice that Citizenry and Delegates have the tendency to decrease from their initial value, obtained with formula (6) and representing the median of their crisp interval, to a value that is less or close to their *lb* (see also Table 7). Decreasing of Citizenry and Delegates to the margin of their crisp interval leads to a second conclusion.

Remark 2: *Participations from citizens and representatives should not be at their highest level, defined by variables of RED 7 that describes the maximum output, in order to obtain the maximum level of E-democracy*.

We conclude this subsection by stating that, after empirically investigating the relativity of E-democracy to inputs scale, the heuristic AOE proves that, with respect to REDs, justice is the most important institution of democratic participation, while citizens and delegates must pay attention to an over-participation.

5.3 The impact of uncertainty on E-democracy

We have already described (section 2) the uncertainty σ as similar to financial volatility. We do not intend to make a parallelism between stochastic processes and fuzzy logic, but we would like to borrow the idea of transforming a field of probability, which must be built on fuzzy logic [38], into a neutral field of probability. E-democracy field must not be risk-neutral but neutral from a generally human mind perception. We have also established a proxy for uncertainty σ, based on RLI and AI, and we create fuzzy subsets with a uniform distribution, using a Gaussian bell-shaped function *gauss2mf*, implemented in Matlab [34].

We discuss in this subsection the influence of uncertainty on E-democracy's outputs. Table 15 presents the influence that σ has on E-democracy, when minimizing.

Table 15. Minimizing E-democracy with σ changing its value for both inputs and output.

No.	x	q	x^*	q^*	σ
1	(0.1 0.15 0.5)	0.05	(0.1 0.15 0.5)	0.05	0.001
2	(0.1 0.15 0.5)	0.050731307	(0.1 0.15 0.5)	0.050731307	0.005
3	(0.1 0.15 0.5)	0.053980545	(0.1 0.15 0.5)	0.053980545	0.01
4	(0.1 0.15 0.5)	0.057321959	(0.1 0.15 0.5)	0.057321959	0.015
5	(0.1 0.15 0.5)	0.060724327	(0.1 0.15 0.5)	0.060724327	0.02
6	(0.1 0.15 0.5)	0.064180304	(0.1 0.15 0.5)	0.064180304	0.025
7	(0.1 0.15 0.5)	0.067683037	(0.1 0.15 0.5)	0.067683037	0.03
8	(0.1 0.15 0.5)	0.071226688	(0.1 0.15 0.5)	0.071226688	0.035
9	(0.1 0.15 0.5)	0.074806324	(0.1 0.15 0.5)	0.074806324	0.04
10	(0.1 0.15 0.5)	0.078417765	(0.1 0.15 0.5)	0.078417765	0.045
11	(0.1 0.15 0.5)	0.082057451	(0.1 0.15 0.5)	0.082057451	0.05
12	(0.1 0.15 0.5)	0.085722333	(0.1 0.15 0.5)	0.085722333	0.055
13	(0.1 0.15 0.5)	0.089409783	(0.1 0.15 0.5)	0.089409783	0.06
14	(0.1 0.15 0.5)	0.093117527	(0.1 0.15 0.5)	0.093117527	0.065
15	(0.1 0.15 0.5)	0.096843586	(0.1 0.15 0.5)	0.096843586	0.07
16	(0.1 0.15 0.5)	0.100586232	(0.1 0.15 0.5)	0.100586232	0.075
17	(0.1 0.15 0.5)	0.104343944	(0.1 0.15 0.5)	0.104343944	0.08
18	(0.1 0.15 0.5)	0.108115384	(0.1 0.15 0.5)	0.108115384	0.085
19	(0.1 0.15 0.5)	0.111899367	(0.1 0.15 0.5)	0.111899367	0.09
20	(0.1 0.15 0.5)	0.115694841	(0.0696 0.15 0.5)	0.115694841	0.095
21	(0.1 0.15 0.5)	0.119500868	(0.0338 0.15 0.5)	0.119500868	0.1
22	(0.1 0.15 0.5)	0.12331661	(0.00235 0.15 0.5)	0.12331661	0.105
23	(0.1 0.15 0.5)	0.127141314	(0.00005 0.15 0.5)	0.127141314	0.11

We use σ = 0.001 instead of σ = 0, in the first line of Table 15, for computational reason; otherwise we transform a Gaussian distribution in a linear one, with a trapezoidal form (i.e. from *gauss2mf* to *trapmf* [34]). Looking at the first 19 observations in Table 15 it seems like the same pattern occurs as in

previous investigations: the initial input vector x and output q are also the optimized solution in case of minimization. This pattern persists for a range of σ from 0.001 to 0.09, but it is not the same for 20th to 23rd observations. Although the results from Table 15 are identical for q and q^*, for a precision 10^{-9} for final output, the difference between first input of x and first input of x^* brings an insignificant modification of the output value.

Table 16. Insignificant changes of E-democracy's output for high values of σ.

σ	q	q^*
0.095	3fbd9e2d559d72**8d**(hex)	3fbd9e2d559d72**5f**(hex)
0.100	0.119500867998**257**	0.119500867998**278**

In order to perceive, for σ = 0.095, the difference between q and q^* with a precision of 10^{-15}, we needed to express the values of both outputs in hexadecimal numbers. For 10^{-15} precision we have, as decimal numbers, the same value (i.e. 0.115694840814692) for both q and q^*. Although larger, an insignificant change of the output also happens for σ = 0.1, but working with a precision of 10^{-14} is hardly probable in social science. Still, Table 16 proves that AOE finds insignificant changes of the output value when this really may occur.

More important is the fact that there is an almost perfect correlation ρ_{min} = 0.999488519, calculated with formula (7), between q^* and σ in Table 16. Thus, if uncertainty goes down so does a possible minimum of E-democracy, which has an interesting interpretation.

Remark 3: *The more certain we are about the non-fuzziness of institutions of democracy the lower could bring us in case of non-democracy*.

In other words, a strict delimitation of good and bad when characterizing institution leads, for a totalitarian society, to worse outputs. On the other hand, when uncertainty is higher, a decrease in the participation of citizens leads to poorer score of E-democracy, see Table 15 and Table 16. However, the change of score is so insignificant and so is the decrease of participation that we must treat this aspect with indifference.

Since we have an optimistic approach, we are more interested in maximization of E-democracy, but a parallel between maximization and minimization of democracy could always give us new perspectives.

Table 17 shows the influence of uncertainty σ on E-democracy in case of maximization, when uncertainty takes the same value for both inputs and output in a range from 0.001 to 0.1.

In case of maximization, the values of initial inputs x is the same in all observations, as for minimization, but the values of q, q^* and x^* change; see Table 15 and Table 17. Thus, we may say that uncertainty σ has a large influence on E-democracy's score. While in case of minimization we have an almost perfect positive correlation, in case of maximization the correlation value between q^* and σ is ρ_{max} = −0.930347942. The linearity is not quite outstanding but the negative value of ρ_{max} proves that when uncertainty increases the score of E-democracy decreases and vice versa. In other words, we have a new conclusion.

Remark 4: *When E-democracy's institutions function well as defined by REDs and variables that define levels of institutions are precisely delimited, so E-democracy may achieve a high level*.

Along with the decrease of uncertainty, representatives are supposed to become more involved in order to achieve a maximum level of democracy. Due to a higher participation from representatives, justice can play a less important role in democracy, but having a value that is still in the range of strong Justice, see Figure 4. Justice becomes more accessible when uncertainty is at a low level (i.e. 0.03) and it appears that justice does not need to compensate anymore the uncertainty when σ < 0.035. This probably occurs because justice can relax its supervision over society if citizens' projection on democracy's institutions is similar.

Table 17. Maximizing E-democracy with σ changing its value for both inputs and output

No.	x	q	x^*	q^*	σ
1	(0.75 0.95 0.5)	0.95	(0.6998 0.8838 0.5)	0.95	0.001
2	(0.75 0.95 0.5)	0.949268692	(0.68175 0.95 0.5)	0.949268692	0.005
3	(0.75 0.95 0.5)	0.946019455	(0.75 0.8972 0.5)	0.946019455	0.01
4	(0.75 0.95 0.5)	0.942678041	(0.75 0.86105 0.5)	0.942678041	0.015
5	(0.75 0.95 0.5)	0.939275673	(0.69885 0.878 0.5)	0.939275673	0.02
6	(0.75 0.95 0.5)	0.935819685	(0.69975 0.95 0.5)	0.935819696	0.025
7	(0.75 0.95 0.5)	0.932314296	(0.7 0.9554 0.5)	0.932316962	0.03
8	(0.75 0.95 0.5)	0.928698361	(0.6999 0.9977 0.5)	0.928773224	0.035
9	(0.75 0.95 0.5)	0.92453815	(0.6999 1 0.45915)	0.925189586	0.04
10	(0.75 0.95 0.5)	0.918683472	(0.6998 1 0.41995)	0.921525155	0.045
11	(0.75 0.95 0.5)	0.909585271	(0.6998 1 0.4)	0.917566159	0.05
12	(0.75 0.95 0.5)	0.896135289	(0.6997 1 0.4)	0.912759997	0.055
13	(0.75 0.95 0.5)	0.878214056	(0.6995 1 0.4)	0.906225818	0.06
14	(0.75 0.95 0.5)	0.856599236	(0.6994 1 0.4)	0.897028211	0.065
15	(0.75 0.95 0.5)	0.832507373	(0.699 1 0.4)	0.884553378	0.07
16	(0.75 0.95 0.5)	0.808360163	(0.698 1 0.4)	0.868756547	0.075
17	(0.75 0.95 0.5)	0.781586728	(0.6971 1 0.4)	0.850164349	0.08
18	(0.75 0.95 0.5)	0.756532684	(0.6959 1 0.4)	0.829680817	0.085
19	(0.75 0.95 0.5)	0.732546579	(0.6959 1 0.4)	0.808325993	0.09
20	(0.75 0.95 0.5)	0.710007981	(0.6944 1 0.4)	0.7870251	0.095
21	(0.75 0.95 0.5)	0.68916955	(0.6925 1 0.4)	0.766493241	0.1
22	(0.75 0.95 0.5)	0.710007981	(0.6944 1 0.4)	0.747205948	0.105
23	(0.75 0.95 0.5)	0.68916955	(0.6925 1 0.4)	0.729422743	0.11

On the other hand, one may say that with the highest uncertainty we reach a score (i.e. $q^* < 0.8$) for E-democracy which is rather *moderate* than *strong*, see Figure 8 and Table 6.

On the account of citizens' participation, it seems that uncertainty does not influence a lot its level, although the value of –0.447429054 shows very little negative correlation, calculated with formula (7), between Citizenry and σ. Still, Citizenry keeps a value around 0.7 with most levels of uncertainty, which gives us important information: participation from citizens must be all the time at a strong level but with some degree of moderation. Actually, all the investigations (see Tables 13, 14 and 17) proved that Citizenry reaches a moderate fuzzy value of strong level (see Figure 7), which entitled us to draw another conclusion.

Remark 5: *In order to achieve a strong E-democracy, citizens should avoid a pure / neat strong participation*.

The reason for having such a moderate - strong Citizenry is the influence of over-participation, see Figure 7 and REDs. The peril of leading E-democracy to a weak level coerces Citizenry to practice a *moderate - strong* activity, instead of a *strong* one, and this does not depend on the level of uncertainty or scale of inputs.

In case of maximization, we have already established that we have a negative correlation between q^* and σ (i.e. $\rho_{max} = -0.930347942$). This is not a very strong linear dependency, but if we look at Table 17 we observe that the third element of x^* (i.e. Delegates) has an important participation to the

maximization of the output for a range of σ from 0.001 to 0.45. The value of ρ_{max} and the behavior of Delegates do tell us that there must be another way of grouping q^* and σ. A way of achieving this is by a cluster analysis, but this is not quite an optimal choice as Figure 13. Correlation between final output q^* and uncertainty σ proves that q^* has values on a specific trend and not scattered at all.

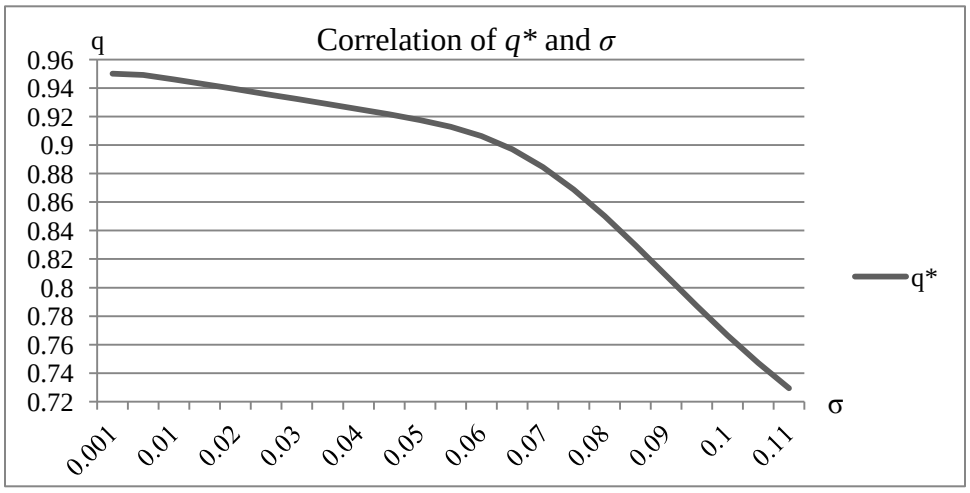

Figure 13. Correlation between final output q^* and uncertainty σ

Instead of a cluster analysis, we propose an affordable approach based on illustration of Figure 13. We split the line of q^* in two trends and we calculate for these trends new coefficients of correlation with formula (7). Figure 14 illustrates how this split is applied for finding an intuitively point of undulation for correlation of q^* and σ.

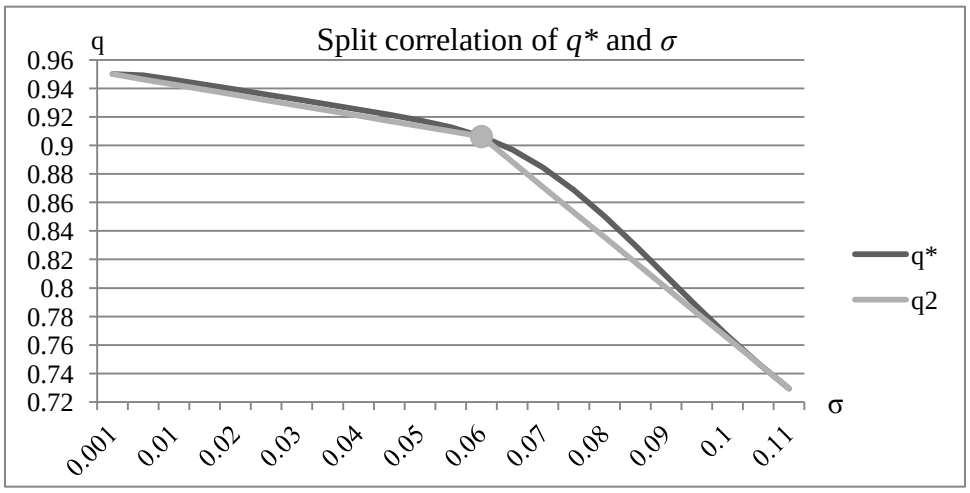

Figure 14. Split correlation between final output q^* and uncertainty σ

What we actually did in Figure 14 was to transform the curve of q^* in the imaginary broken line of *q2*, which has two straight lines. In Figure 14 we present choices of grouping the 23 observations from Table 17 in two cluster by calculating their coefficient of correlation with formula (7).

Table 18. Different choices for splitting q^* curve in a broken line

Option	**σ = 0.045**		**σ = 0.05**		**σ = 0.055**		**σ = 0.06**		**σ = 0.065**	
Position	1-10	11-23	1-11	12-23	1-12	13-23	1-13	14-23	1-14	15-23
$\rho(q^*, \sigma)$	-0.9981	-0.9923	-0.9981	-0.9923	-0.9976	-0.9961	-0.9951	-0.9984	-0.9882	-0.9995

Taking into consideration data from Table 18, we may choose our splitting point on the curve of q^* somewhere around the value of $\sigma = 0.06$, which is not quite far from the point of undulation intuitively chosen in Figure 14.

This analysis is not very complicated, but it rather proves that E-democracy's outputs depend on uncertainty somehow on two linear trends. One trend deals with low uncertainty and high values of E-democracy and the other explains the behavior of higher uncertainty and lower scores of E-democracy.

Another approach is to find a way of splitting more accurate, constructing an imaginary broken line over the curve of q^* with three straight line. Thus, we illustrate in Figure 15 a three trends correlation of q^* and σ, given by the broken line of $q3$.

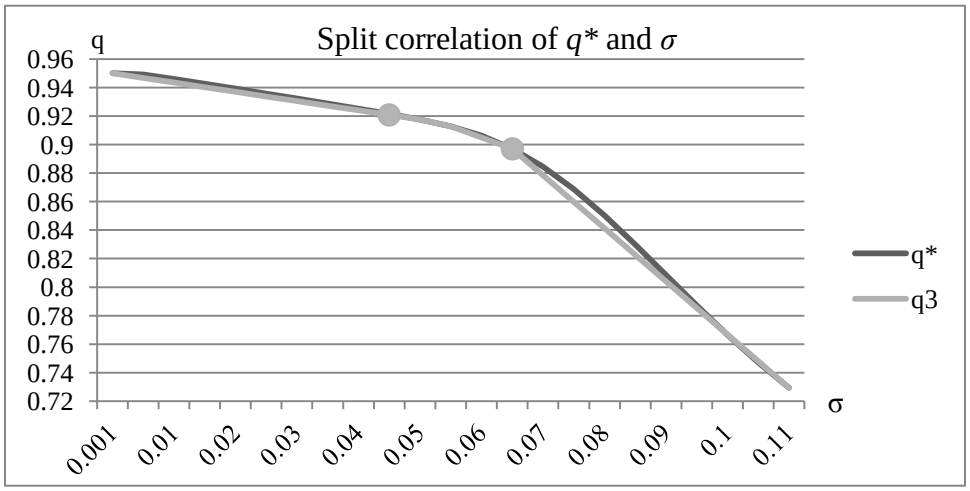

Figure 15. Split correlation between q^* and σ by three clusters

Table 19 presents the values of the 23 observation from Table 17 grouped in the three clusters of Figure 15, based on coefficient of correlation calculated with formula (7).

Table 19. Splitting q^* curve in a broken line composed of three straight lines

Option	**$\sigma \leq 0.05$**	**$0.055 \leq \sigma \leq 0.065$**	**$\sigma \leq 0.07$**
Position	1-11	12-14	15-23
$\rho(q^*, \sigma)$	-0.9981	-0.9953	-0.9995

As final remarks of this subsection, we discuss the division of σ values in two or three cluster, using correlation with final output q^*. We can split E-democracy's scores either in two groups, see Table 18 and Figure 14, or in three groups, see Table 19 and Figure 15. We believe that the approach with three clusters is more accurate and, thus, we can split E-democracy's score with respect to uncertainty in low, moderate and high output. This gives an easier perspective on E-democracy's outputs, also establishing that the level of E-democracy would be (theoretically) moderate at this moment, taking into consideration the implied value of uncertainty $\sigma = 0.065$ and, more important, with respect to REDs. We remind that this implied value of $\sigma = 0.065$ is a proxy obtained with RLI and AI, see section 2. On the other hand, we could have used a nonlinear regression between q^* and σ with a semi-logarithmic and logarithmic approach or classic technique of clustering or even fuzzy clustering. These techniques did not show any concluding result and they are too complex and toilsome to be presented at this very moment, but further investigations on more precise values of σ could be subject of this kind of research.

5.4 Boundaries of inputs and E-democracy's output

We have already discussed the infinity of solutions for optimum input that yields an optimum output, see Appendix I. We have established that the output is unique, but there is not just one input that

leads to the best result. Starting from the final solution x^* obtained with AOE, we will find the bounds for each element when all the other elements of x^* are fixed (i.e. calculating bounds of Citizenry when Justice and Delegates are fixed). Without fixing all the other elements it would be too difficult to handle the multiple fluctuations of inputs.

We briefly present the algorithm that finds boundaries for each input (AIB) when all the other inputs are fixed. AIB is a bisection method extension and a Matlab implementation of AIB is provided in Appendix III.

AIB 1) finding an initial lower bound (l) and initial upper bound (u) for $x_i^* = 1$, n ($n = 3$ in this case);

AIB 2) applying bisection method for both intervals: (l, x_i^*) and (x_i^*, u);

AIB 3) stop when mfis(l) and mfis(u) < mfis(x_i^*) for maximization and mfis(l) and mfis(u) > mfis(x_i^*) for minimization (formula (5) presents function mfis).

When minimizing E-democracy's output, with respect to levels of uncertainty, bounds of final inputs (BFI) are all the same for most values of σ. The vector of type x of lower BFI (LBFI) is (0;0;0) and for upper BFI (UBFI) is (1;1;1). So, if one of the Justice or Citizenry reaches its lowest level all the other two inputs (i.e. Citizenry and Delegates or, respectively, Justice and Delegates) can fluctuate from 0 to 1, which means that they can take any value no matter the degree of uncertainty.

Table 20 presents exceptions from UBFI for some values of σ, using a precision of 0.0001 for minimized output q^*, otherwise infinitesimal changes in the output would influence the search for boundaries, see also Table 16.

Table 20. Exceptions of UBFI for high values of uncertainty σ

σ	*LBFI*	x	*UBFI*	q^*
	0	0.1000	0.2000	
0.0900	0	0.1500	1.0000	0.1119
	0	0.5000	1.0000	
	0	0.0696	0.1896	
0.0950	0	0.1500	1.0000	0.1157
	0	0.5000	1.0000	
	0	0.0338	0.1988	
0.1000	0	0.1500	1.0000	0.1195
	0	0.5000	1.0000	
	0	0.0024	0.1824	
0.0105	0	0.1500	1.0000	0.1233
	0	0.5000	1.0000	
	0	0.0001	0.2000	
0.0110	0	0.1500	1.0000	0.1271
	0	0.5000	1.0000	

In case of minimization, we have already established the value of correlation between σ and q^* is ρ_{min} = 0.999488519, see subsection 5.3. On the contrary, correlation between inputs or any input BFI and σ is weak (i.e. 0.41 for Citizenry UBFI and σ). On the other hand, the only input that does not stretch from 0 to 1 is Citizenry, when minimizing with high values of σ. This is important because it tells us that a minimum level of E-democracy can be surpassed with a value little higher than 0.2 for Citizenry, but not by Justice or Delegates, when the other two inputs are fixed.

Table shows some results of E-democracy when Citizenry and Justice change their values for a constant uncertainty σ = 0.095 (Delegates has a conventional 0.5 value, practically indifferent when minimizing, see RED 1).

Table 21. Non E-democracy from several perspectives

Changed Input	x	q
-	(0.1000 0.15 0.5000)	0.1119
1	(0.2100 0.15 0.5000)	0.1121
1	(0.7100 0.15 0.5000)	0.1121
2	(0.1000 1 0.5000)	0.1119
3	(0.1000 0.15 1.0000)	0.1119
2,3	(0.1000 1 1.0000)	0.1119
1,2	(0.2100 0.16 0.5000)	0.1122
1,2	(0.3100 0.36 0.5000)	0.2266
1,2	(0.5100 0.56 0.5000)	0.3481
1,2	(0.7100 0.56 0.5000)	0.3481
1,2	(0.7100 0.76 0.5000)	0.4565
1,2	(0.7100 0.96 0.5000)	0.7946

It is clear that, for a high level of uncertainty, a change in Justice and/or Delegates does not bring an improvement higher than precision 0.0001. Only Citizenry can bring a slight improvement on E-democracy's score, but this slight change is poor when Citizenry acts alone. Only acting together with Justice, Citizenry improves E-democracy's score. We may now draw another important conclusion.

Remark 6: *In order to build democracy, only citizens can make a first step, but, without justice, only an infinitesimal change of the output can be achieved*.

From a certain incipient point of evolution, only justice can bring democracy to a high level. This happens when uncertainty is high, exactly as in moments when primitive societies create their state from a bottom-up approach [6].

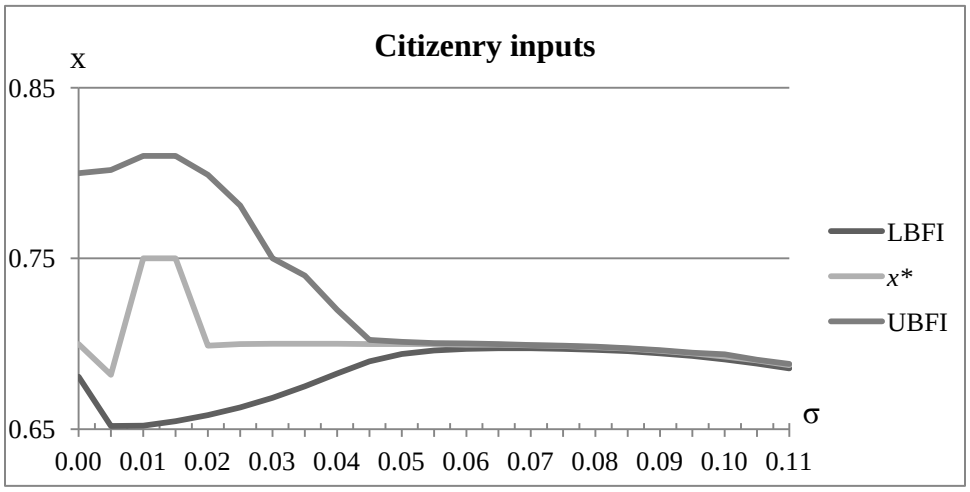

Figure 16. Citizenry's x* and BFI with respect to uncertainty when maximizing E-democracy

Things are a quite different when maximizing E-democracy and, for a range from 0.05 to 0.1 for uncertainty σ, there is no fluctuation for any input when the other two are fixed, without using a precision 0.0001. Thus, for these levels of uncertainty, the maximized inputs do not have any LBFI and

UBFI, both BFI are not distinct from final value of inputs if we choose to use a precision higher than 10^{-4}. However, we believe that a precision of 10^{-4} is a solid one when dealing with social sciences, like in our case.

Figure 16 illustrates behavior of Citizenry's final values and its BFIs with respect to uncertainty σ, using the same range of discrete values from 0.001 to 0.11, when maximizing E-democracy. We notice in Figure 16, the irregularity of Citizenry's final values interval, which becomes narrower with $\sigma > 0.05$. When $\sigma < 0.05$, Citizenry can take values from a wider interval when Justice and Delegates are fixed, in order to obtain the final output q^x; for a precision of 10^{-4} $q^x \in (q^* - 10^{-4}, q^* + 10^{-4})$. Thus, for high uncertainties Citizenry must remain moderate - strong, a quantified level of participation of 0.7, with a slight descendent trend for ascending σ. For low values of uncertainty, Citizenry can fluctuate from moderate - strong to strong and even strong - over (see Figure 7). This leads us to another conclusion.

Remark 7: *Citizens can become more involved in political process when projection of society's members over its institutions is quite similar*.

Otherwise, citizens must remain to a narrower range of options, even if we quantify E-democracy's score with low precision (i.e. 10^{-4}).

Figure 17 explains behavior of Justice, when Citizenry and Delegates are fixed, and we use the same precision 10^{-4} for final outputs and inputs, with respect to uncertainty σ, when maximizing E-democracy.

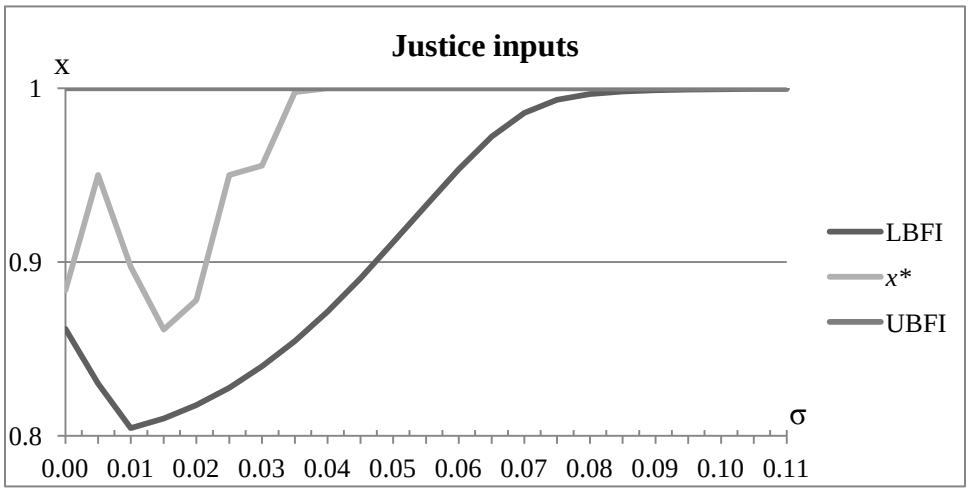

Figure 17. Justice's *x and BFI with respect to uncertainty when maximizing E-democracy**

It is clear once again, as Figure 17 proves it, that Justice tends to its maximum possible value (i.e. 1 on 0-1 scale). We observe a similar behavior for Justice, comparing to Citizenry. While for low values of uncertainty it has a wider interval that leads to best score of E-democracy, for high values of σ Justice and its BFIs converge to maximum possible value of 1. Thus, Justice can fluctuate from a moderate - strong level to a strong level (see also Figure 4). We take this as a new conclusion.

Remark 8: *A relaxation of demands from justice occurs when members of society look at society's problem in an almost identical way*.

Figure 18 illustrates behavior of Delegates, when Citizenry and Justice are fixed, for a precision of 10^{-4} for final input value, with respect to uncertainty σ, when maximizing E-democracy.

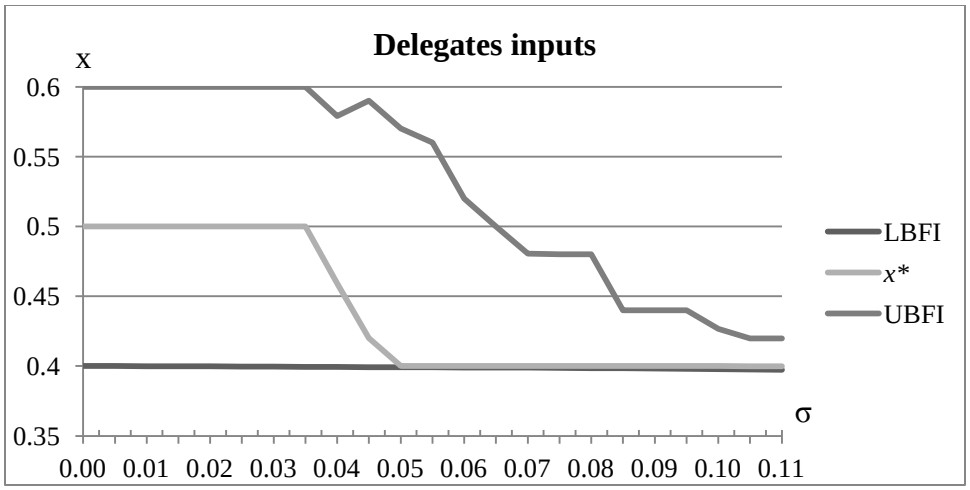

Figure 18. Delegates' *x and BFI with respect to uncertainty when maximizing E-democracy**

We notice the same pattern for low values of uncertainty for Delegates, comparing to Citizenry and Justice. We also observe a convergence value for Delegates and its BFIs, but not as precisely as for Citizenry and Justice. While, for low values of uncertainty, Delegates has the possibility to fluctuate from a low-moderate level to a moderate one (see Figure 6. FD), for high values of σ Delegates and its LBFI converge to the minimum crisp value of a moderate level (i.e. lb = 0.4), although it may get under with a few commas. Delegates' UBFI also tends to converge toward moderate minimum crisp value, but not as faster as Delegates' x^* and its LBFI. Practically, Delegates' LBFI is stuck to moderate lb, which should be taken as a moderate involvement from elected representatives when uncertainty is high.

Remark 9: *Despite many believes, representatives cannot improve E-democracy's score when citizens' perception over democracy's institutions is not oriented on the same direction, with respect to REDs.*

On the contrary, investing with power and delegating authority to representative body will not lead to a better participative society; this happens only when system uncertainty is low, but there are also the peril of non-democracy, see **Remark 3**.

Table 22 presents some results of minimization and maximization for a range of uncertainty σ from 0.001 to 0.11.

Table 22. E-democracy's score with respect to uncertainty

σ	Minimization		Maximization	
	x^*	q^*	x^*	q^*
0.001	(0.1000 0.15 0.5)	0.05	(0.6998 0.8838 0.5000)	0.95
0.005	(0.1000 0.15 0.5)	0.0507	(0.6807 0.8614 0.4000)	0.9493
0.04	(0.1000 0.15 0.5)	0.0748	(0.7199 1 0.5792)	0.9252
0.065	(0.1000 0.15 0.5)	0.0931	(0.6994 1 0.4000)	0.897
0.08	(0.1000 0.15 0.5)	0.1043	(0.6983 1 0.4800)	0.8502
0.095	(0.0696 0.15 0.5)	0.1157	(0.6944 1 0.4000)	0.787
0.11	(0.0001 0.15 0.5)	0.1271	(0.6875 1 0.3999)	0.7294

We notice in Table 22 that we have a sort of symmetry of minimization and maximization of E-democracy, when it comes to minimum and maximum possible values. For σ = 0.001 (almost zero, for computational reason) the score of E-democracy in case of minimization is q^*_{min} = 0.05 and in case of maximization is q^*_{max} = 0.95. Both have the same absolute deviation 0.05 from the margins of the interval that define E-democracy's scale (i.e. 0 and 1). This tells us two important things:

Remark 10: *With respect to REDs, we cannot reach a possible minimum (i.e.* 0 *for* 0-1 *scale) or a possible maximum (i.e.* 1 *for* 0-1 *scale) for E-democracy's score.*

Remark 11: *More, the lack of uncertainty (i.e.* σ = 0.001*) could lead us to worst result in case of non-democracy or to the best score of E-democracy.*

Another interesting observation must be made when it comes to fuzziness of E-democracy and the relation between minimum and maximum outputs (i.e. q^*_{min} and, respectively, q^*_{max}), with respect to uncertainty σ. If we calculate the sum of q^*_{min} and q^*_{max} from Table 15 and Table 17, we notice that the result is *sum* (q^*_{min}, q^*_{max}) = s = 1, for low values of uncertainty.

Table 23 presents, for different ranges of uncertainty, results that help establishing some relations concerning the fuzziness of E-democracy.

Table 23. Fuzziness of E-democracy with respect to range of uncertainty σ

σ range	q^*_{min}	q^*_{max}	s range	$\rho(\sigma, s)$
0.001-0.03	0.05-0.0677	0.95-0.9323	1	-0.6196
0.035-0.065	0.0712-0.0931	0.9288-0.8970	0.999-0.990	-0.8333
0.07-0.11	0.0968-0.1271	0.8846-0.7294	0.9814-0.8566	-0.9992

For 0.001 ≤ σ ≤ 0.03, we may say that E-democracy is a non-fuzzy system, when it comes to optimum results (i.e. q^*_{min} and q^*_{max}), because s = 1. Taking into consideration the 0-1 scale used to build the fuzzy system of E-democracy, we notice that 1 – q^*_{min} = q^*_{max} and this proves that low uncertainty transforms E-democracy in a crisp bivalent logical system, see first line of observations in Table 23.

For 0.035 ≤ σ ≤ 0.065, E-democracy's optimum outputs leave behind non-fuzziness, in an ascending trend for σ, see second line of observations in Table 23.

The last line of observations in Table 23, for 0.07 ≤ σ ≤ 0.11, not only offers an evident fuzzy E-democracy, from optimum outputs point of view, but it also shows an almost perfect correlation between σ and s, calculated with formula (7). These trends, in the three clusters of uncertainty values from Table 23, lead to the last conclusion of this exploration.

Remark 12: *High levels of uncertainty determine a fuzzy E-democracy's model, with higher values for minimum output and lower values for maximum output. Low levels of uncertainty describe a non-fuzzy E-democracy's model, with lower values for minimum output and higher values for maximum output.*

Figure 19 illustrates the range of E-democracy's outputs, with respect to uncertainty σ.

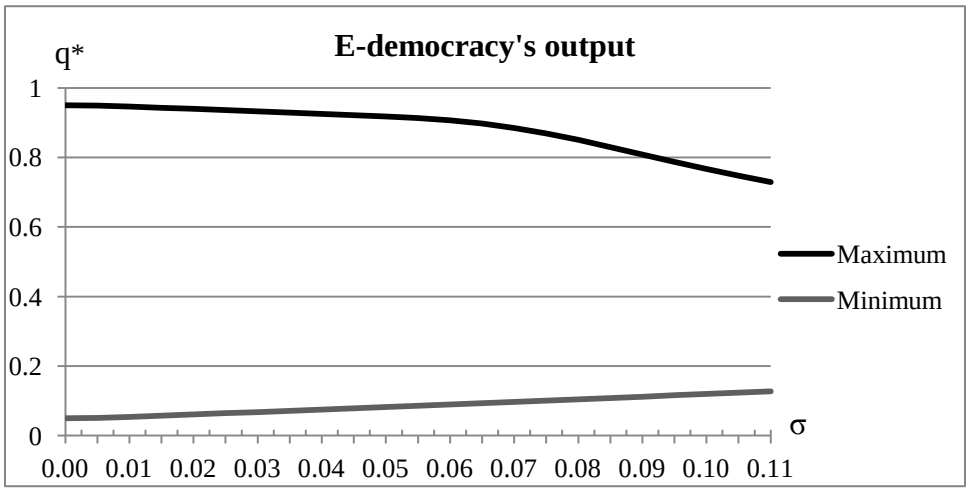

Figure 19. E-democracy's score for minimization and maximization with respect to σ

We have already established that the correlation between minimized output q^*_{min} and uncertainty σ is $\rho(q^*_{min}, \sigma)$ = 0.999488519, and this linearity is proved by Figure 19. Based on coefficient of correlation,

calculated with formula (7), we have $\rho(q^*_{max}, \sigma) = -0.930347942$, and Figure 19 illustrates this low level of linearity. From a certain point (i.e. $\sigma = 0.035$) the descending trend of q^*_{max} is more pronounced than the ascending trend of q^*_{min}, with respect to uncertainty σ and, thus, the fuzziness of E-democracy's model becomes more intelligible, see also Table 23.

Conclusions

Liberal or representative democracy [20, 43] has come to an end of its evolution and we cannot expect major changes. There is nothing to be blamed on this type of democracy; it has done, when rule of law established, the best of what it should have done. Problems appear because of citizens and not because of their elected delegates. It is the refusal of people to participate with substantial effort to political process, and this participation may not be mistaken for a poor democratic expression such as voting. Deliberation is one of the most important institutions of democracy, and we believe that democracy has both a non-instrumental and an instrumental role. Democracy must be practiced continuously, as a way of living especially from a social, economic and cultural point of view. Daily political manifestations of non-elected citizen are not an option, but one must be ready to exercise one's democratic right of participation. Citizen's participation is important from two points of view: *i*) the need for constantly learning and improving democracy and *ii*) to put pressure on representatives, by sending the message that justice is not alone in watching political process. Justice is above citizens and other democratic institutions, but it must itself be subject of reinventing. A strong form of justice is a justice that is malleable to new scientific discoveries or to new beliefs arisen from inclusion of others, deliberations and/or new paradigms [7]. On the other hand, we must pay attention that citizenry and justice do not become instruments of non-democratic attitudes that have people's endorsement and juridical alibi, such as 1930s rise of Nazism [53] or 2012 political situations in Romania, a European Union country [54]. We are not naïve to believe that cultural or scientific research will lead to the transformation from representative democracy to a participative democracy. Neither citizens will claim their right to become politically active on their own [55] nor without a common interest of elites, to protect themselves from economic burdens, will see any transformation [56]. If there is a chance given to change, this consists in education of children, and we can hopefully expect for a future better society, which foundation must be done today.

In this paper, we have been trying to explore, from a more appropriate logic of a society of inclusion and diversity, what the outcomes of E-democracy are relative to a given scale, with respect to pre-defined rules (i.e. REDs). We used an algorithm (AOE) that finds an optimum result and the optimum inputs for E-democracy; see Appendix I for details. Based on AOE, we have empirically proved that the scale of inputs and output is not important, but uncertainty is. We also advanced a proxy, determined with RLI [32], for uncertainty that defines fuzzy sets of inputs and output.

With respect to eight fuzzy rules (REDs) we made some observations and drew some conclusions. The first and most important one is that justice must be at a high level if we search for an optimum E-democracy (**Remark 1**). Participations of citizens must pay attention to over-activity (**Remark 5**) and should stay at a constant moderate - strong level and so must do delegates, only for low - moderate level (**Remark 2**). Low uncertainty about the levels of E-democracy's institutions may lead to minimum or maximum outcome (**Remark 3**, **Remark 4** and **Remark 11**). The involvement of citizens may become more pronounced in the same conditions of low uncertainty of E-democracy's model (**Remark 7**). Only citizens can start the process of building a democratic society, but there is impossible to develop it without justice (**Remark 6**). On the other hand, representatives may become more involved (**Remark 9**) and demands from justice may be reduced (**Remark 8**) when uncertainty of E-democracy's model is low. However, low uncertainty describes a non-fuzzy E-democracy's model (**Remark 12**). Democracy has

been always considered as the most desired system by opponents of totalitarian and authoritarian models of society, but never considered as perfect, which is proved even for E-democracy. Even when uncertainty is almost zero and projection over institutions of E-democracy's model is identical, the maximum output is not the theoretical possible maximum (**Remark 10**).

Future research must verify possibilities of new proxies for uncertainty or ways to quantify this uncertainty, which is the most important element of our model of E-democracy. On the other hand, by calculating the values for minimum and maximum inputs that yield the optimum outputs, we only established some theoretical measures of E-democracy's institutions. More appropriate is to find instruments, based on mathematical, sociological and/or statistical models, to discover what the real levels of E-democracy's inputs are. Then, we could compare the real values of E-democracy's inputs with the theoretical ones, in order to adjust them for an optimum (hopefully, a maximum) result.

Acknowledgement

This work was cofinanced from the European Social Fund through Sectoral Operational Programme Human Resources Development 2007-2013, project number POSDRU/159/1.5/S/134197 „Performance and excellence in doctoral and postdoctoral research in Romanian economics science domain".

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Appendix

I. AOE: http://turcoane.com/fss/AOE.pdf
II. AOE-IAOE: http://turcoane.com/fss/aoe_iaoe.txt
III. AIB: http://turcoane.com/fss/AIB.txt